SAT Math Workbook

STRATEGIC
Test Prep

Table of Contents

Chapter 1
Intro to the Math Modules

Introduction👋

SAT Math Modules Layout

It is important to understand the layout of the Math Sections, as well as the types of questions involved.

Module 1 - 22 Questions, 35 minutes
Module 2* - 22 Questions, 35 minutes
*This module may be harder or easier depending on how you did on Module 1.

Categories & Question Types

1. **Algebra (13-15 questions)**
    - Linear equations in 1 variable
    - Linear equations in 2 variables
    - Linear functions
    - Systems of 2 linear equations in 2 variables
    - Linear inequalities in 1 or 2 variables
2. **Advanced Math (13-15 questions)**
    - Equivalent expressions
    - Nonlinear equations in 1 variable
    - Systems of equations in 2 variables
    - Nonlinear functions
3. **Problem-Solving and Data Analysis (5-7 questions)**
    - Ratios, rates, proportional relationships, and units
    - Percentages
    - One-variable data: distributions and measures of center and spread
    - Two-variable data: models and scatterplots
    - Probability and conditional probability
    - Inference from sample statistics and margin of error
    - Evaluating statistical claims: observational studies and experiments
4. **Geometry & Trigonometry (5-7 questions)**
    - Area and volume formulas
    - Lines, angles, and triangles
    - Right triangles and trigonometry
    - Circles

SAT Math Strategies 

General Strategy	What It Is/Example	When You Use It
Christmas Tree		Running out of time
Leave a Trail		On All Modules
1-Minute Rule		On All Modules
Math Strategy	**What It Is/Example**	**When You Use It**
Put '3'		
Pick a Middle #		
Majority Rules		
Work Backwards		
Redo Every Problem		
Draw a Picture		
Desmos!	You will see :)	You will see :)

Chapter 2

Using Desmos

Using Desmos

Desmos is an amazing tool built into BlueBook that shows you the right answers quickly. When are some good times to use Desmos?

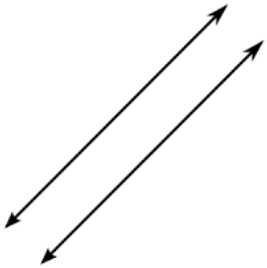
- Systems of Equations or inequalities (when you have 2 or more equations/inequalities)
- Problems that have ONE a, b, and/or c constant in them (use a slider)
- Complicated-looking Algebra problems
- When given at least two points and you know the equation (regression)

Desmos Concept #1: Systems of Equations

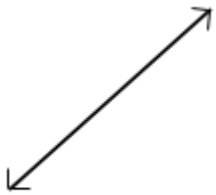
Determining how many solutions a system has is now easier than ever on the DSAT.

💡 **Tip:** Type both equations into Desmos.

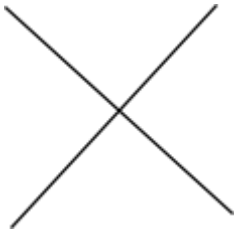
Parallel Lines: have _____ solutions



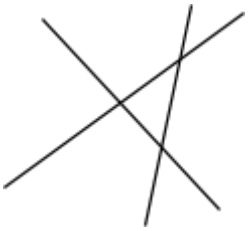
One Line: have _____ solutions



Two Intersecting Lines:
have _____ solutions



How many solutions does this system of 3 lines have? ____



What about a Quadratic (Parabola) and Line System?

# of Intersecting Points	**# of Solutions**
0 Intersections	
1 Intersection	
2 Intersections	

DESMOS SYSTEMS PRACTICE PROBLEMS

Question 1:

$$y = 3x + 5$$

$$2y = 7x + 2$$

The solution to the given system of equations is (x, y). What is the value of $x - y$?

Question 2:

$$\frac{9}{2}x + 4y = 27$$

$$\frac{3}{2}x + 4y = 21$$

The solution to the given system of equations is (x, y). What is the value of

$$\frac{17}{2}x + 6y\,?$$

A) 24
B) 36
C) 44
D) 57

Question 3:

$$2x - 5y = 5y + 1$$

$$hy = 3 + 2x$$

In the given system of equations, h is a constant. If the system has no solution, what is the value of h?

A) 2
B) 5
C) 10
D) 15

Question 4:

$$y = x - c$$

$$y = -(x - 5)^2$$

In the given system of equations, c is a constant. The system has two distinct real solutions. Which of the following could be the value of c?

A) 4
B) $\frac{19}{4}$
C) 5
D) 2

Question 5:

$$y + k = 2x$$

$$y - k = x^2 - 6x$$

In the given system of equations, k is a constant. The system has exactly one distinct real solution. What is the value of k?

Question 6:

$$4x + 7y = 10$$

$$8x + 14y = 20$$

For each real number r, which of the following points (x, y) lies on the graph of both equations in the xy-plane?

A) $\left(r, \frac{4r-10}{7}\right)$

B) $\left(r, \frac{10-4r}{7}\right)$

C) $\left(\frac{7r-10}{4}, r\right)$

D) $\left(\frac{6-7r}{4}, r\right)$

Question 7:

$$3x + 2y = 6$$

$$6x + 4y = 12$$

For each real number r, which of the following points (x, y) lies on the graph of **both** equations in the xy-plane for the given system?

A) $\left(r, 3 - \frac{3}{2}r\right)$

B) $\left(r, 3 - \frac{r}{2}\right)$

C) $\left(3 - r, 3 - \frac{3}{2}r\right)$

D) $\left(3, 2 - \frac{3}{2}r\right)$

DESMOS CONCEPT #1 SYSTEMS OF EQUATIONS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

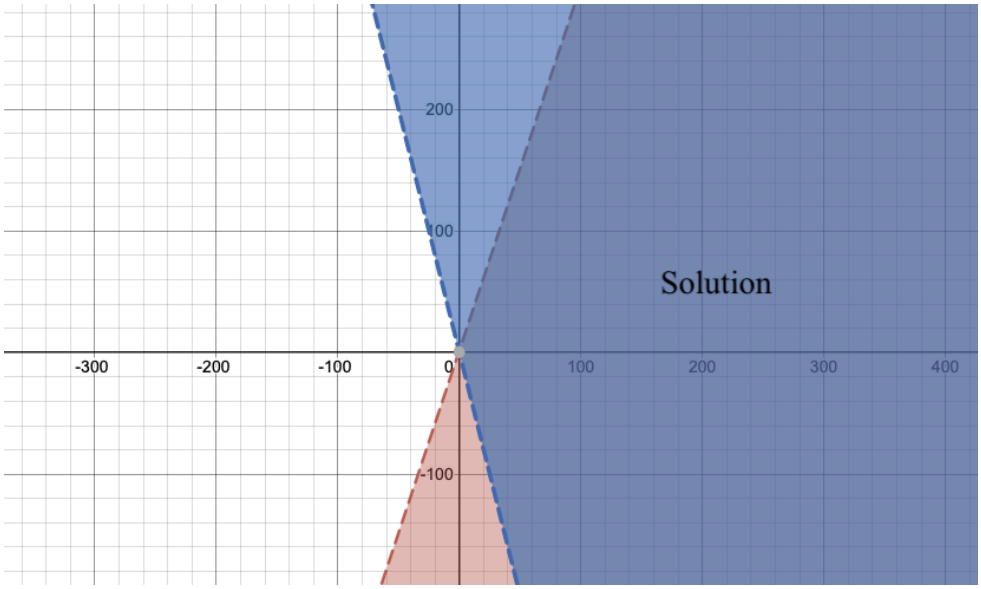
Date completed: _____

- ☐ **1. -21**
- ☐ **2. C**
- ☐ **3. C**
- ☐ **4. C**
- ☐ **5. 8**
- ☐ **6. B**
- ☐ **7. A**

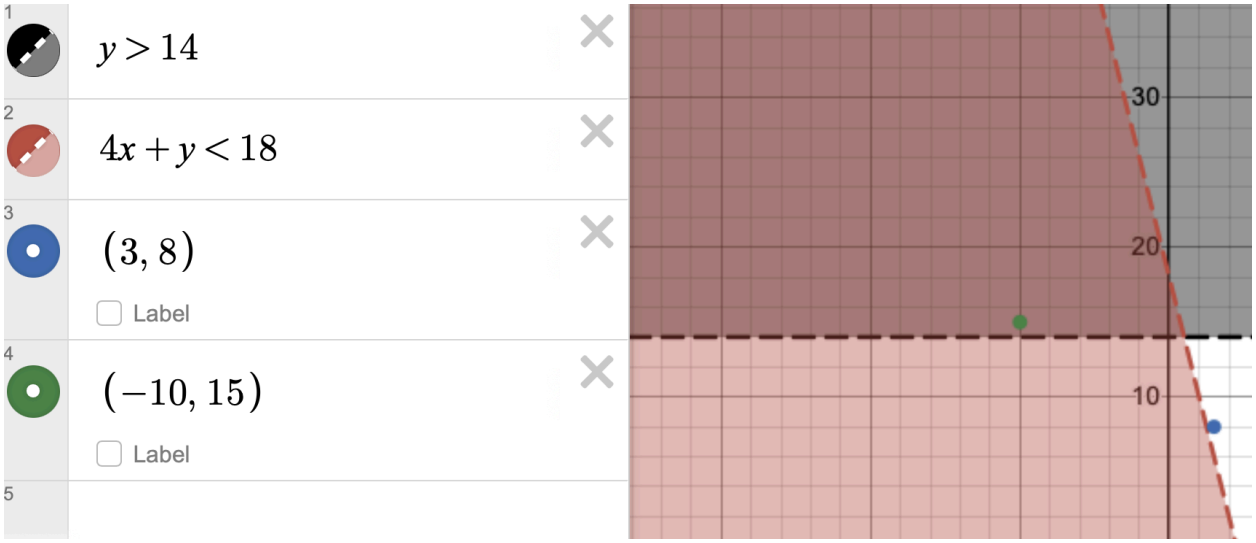
Desmos Concept #2: Inequalities

WHAT YOU NEED TO KNOW

The solution set is the darkest region (where the two solutions overlap)



You can type coordinates in underneath your inequalities and make sure the colored dots are in the solution:



DESMOS INEQUALITIES PRACTICE PROBLEMS

Question 1:

$$y < -\frac{2}{3}x + 4$$

$$y > \frac{2}{3}x - 2$$

Which point (x, y) is a solution to the given system of inequalities in the xy-plane?

A) $(6, 0)$
B) $(0, -2)$
C) $(3, 5)$
D) $(-3, 2)$

Question 2:

$$y > 20$$

$$3x + y < 25$$

The point $(x, 42)$ is a solution to the system of inequalities in the xy-plane. Which of the following could be the value of x?

A) -8
B) -4
C) 6
D) 10

Question 3:

$$y < -3x + 5,$$

$$y > 2x - 1$$

Which of the following tables contains **only** values of x and their corresponding y values that satisfy **both** inequalities?

A)

x	y
-1	3
0	5
2	1

B)

x	y
-2	8
0	3
-3	-2

C)

x	y
-1	4
1	3
2	-1

D)

x	y
-1	6
1	1
2	-2

Question 4:

(Hint: Before typing in Desmos, make sure to use x and y, not some other variables like s and p)

A bakery owner budgets **$4,800** to purchase flour. The owner must purchase at least **300** bags of flour to qualify for a bulk discount. If the owner pays **$10** per bag for standard flour and **$20** per bag for premium flour, what is the maximum number of premium flour bags the owner can purchase while staying within the budget and maintaining the bulk discount?

DESMOS CONCEPT #2 INEQUALITIES ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. D**
- ☐ **2. A**
- ☐ **3. B**
- ☐ **4. 180**

Desmos Concept #3: Regressions

With Regressions, Desmos can figure out the equation for you.

When is it a good time to use a regression?

✅ When they give you points on the function and ask you to come up with the equation

✅ Typically when you see a table of values for x, y and they want an equation

Tips:

💡You have to use X1, Y1 when making your equations (x and y won't work)

💡You have to use ~ instead of =

💡You need to know your equation formats, like a line is y=mx+b

DESMOS REGRESSION PRACTICE PROBLEMS

Question 1:

Points (-1, 4) and (-2, 6) lie on a line. What is the y-intercept?

Question 2:

The function h is defined by

$$h(x) = a^x + b,$$

where a and b are positive constants. The graph of $y = h(x)$ in the xy-plane passes through the points $(0, 6)$ and $(3, 32)$. What is the value of ab?

Question 3:

x	$h(x)$
-18	3
-6	5
6	1

The table shows three values of x and their corresponding values of $h(x)$, where

$$h(x) = \frac{f(x)}{x + 2}$$

and f is a linear function. What is the y-intercept of the graph of $y = f(x)$ in the xy-plane?

A) $(0, 30)$

B) $(0, 10)$

C) $(0, 5)$

D) $(0, -6)$

Question 4:

x	$h(x)$
-20	4
-15	12
-10	36

The table shows three values of x and their corresponding values of $h(x)$, where

$$h(x) = \frac{f(x)}{x+5}$$

and f is a linear function. What is the y-intercept of the graph of $y = f(x)$ in the xy-plane?

A) $(0, -270)$

B) $(0, -290)$

C) $(0, -295)$

D) $(0, -300)$

Question 5:

The function h is defined by $h(x) = a^x + b$, where a and b are positive constants. The graph of $y = h(x)$ in the xy-plane passes through the points $(0, 4)$ and $(2, 12)$. What is the value of ab?

A) **8**

B) **9**

C) **12**

D) **15**

DESMOS CONCEPT #3 REGRESSION ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. 2**
- ☐ **2. 15**
- ☐ **3. D**
- ☐ **4. D**
- ☐ **5. B**

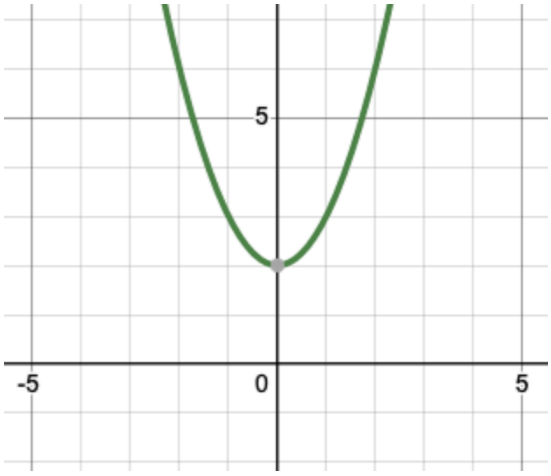
Desmos Concept #4: Quadratics

WHAT YOU NEED TO KNOW

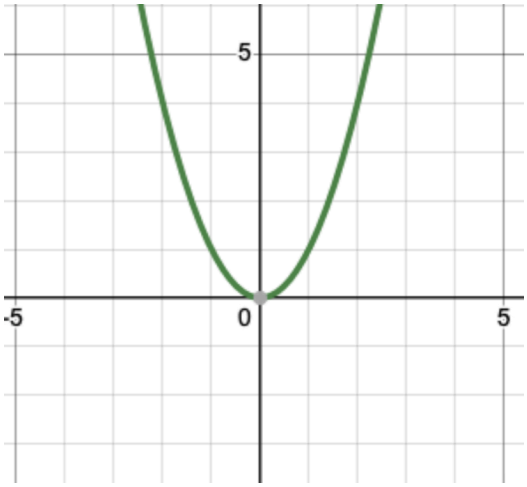
Standard Form (for regressions): _____

Vertex Form (for regressions): _____

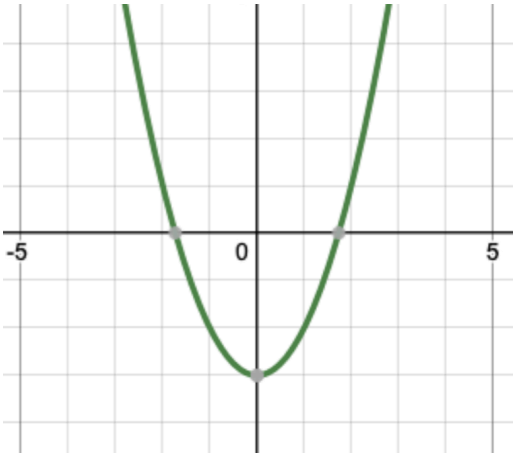
0 Distinct Solutions



1 Distinct Solution

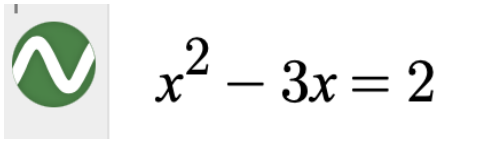


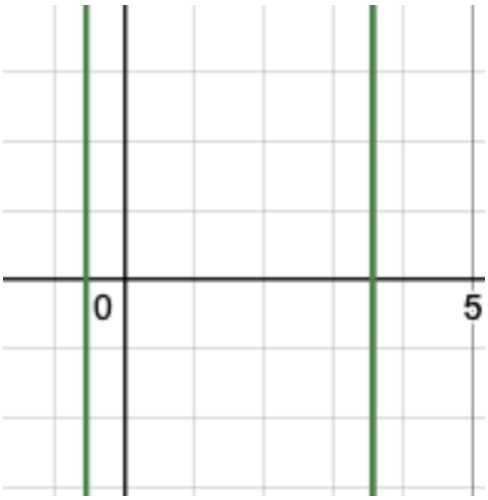
2 Distinct Solutions



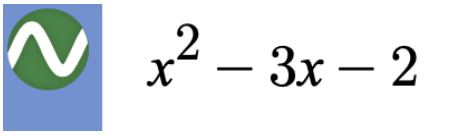
💡**Tip:** Move everything to one side first and avoid using an equal sign before putting a quadratic into Desmos. If you enter it with an equal sign, you will get lines straight up and down.

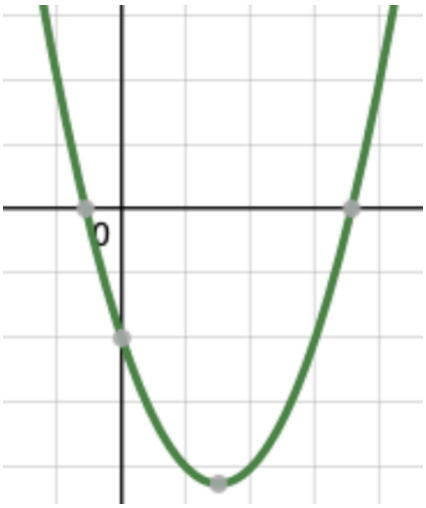
Example with Equal Sign





Example without Equal Sign





DESMOS QUADRATICS PRACTICE PROBLEMS

Question 1:

Which quadratic equation has **no** real solutions?

A) $x^2 + 8x - 16 = 0$

B) $x^2 + 8x + 16 = 0$

C) $2x^2 + 8x + 16 = 0$

D) $2x^2 + 8x - 16 = 0$

Question 2:

$$y = a(x - 1)(x + 2)$$

In the quadratic equation above, a is a nonzero constant. The graph of the equation in the xy-plane is a parabola with vertex (h, k). Which of the following is equal to k?

A) $-a$

B) $-\frac{9a}{4}$

C) $-\frac{5a}{2}$

D) $-2a$

Question 3 (Hint: Regression):

The function f is a quadratic function. In the xy-plane, the graph of $y = f(x)$ has a vertex at $(2, 4)$ and passes through the points $(4, 16)$ and $(0, 20)$. What is the value of $f(-1) - f(3)$?

Question 4:

$$2x^2 = 4 - 6x$$

What is the x-coordinate of the vertex of the parabola?

Question 5 (Regression):

The table below shows three values of x and their corresponding values of $f(x)$ for a quadratic function f. Based on these points, which equation defines f?

x	$f(x)$
-1	2
0	1
2	11

A) $f(x) = 2x^2 + x + 1$
B) $f(x) = x^2 + x + 1$
C) $f(x) = 2x^2 + 2x + 1$
D) $f(x) = 2x^2 + x - 1$

DESMOS CONCEPT #4 QUADRATICS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. C**
- ☐ **2. B**
- ☐ **3. 32**
- ☐ **4. 1.5**
- ☐ **5. A**

Desmos Concept #5: Circle Equation

WHAT YOU NEED TO KNOW

1. A circle equation is in the form
$(x - h)^2 + (y - k)^2 = r^2$, *where* (h, k) *is the center and* r *is the radius.*

Example: $(x - 2)^2 + (y + 5)^2 = 16$

center: (____, ____)

radius:

DESMOS CIRCLE PRACTICE PROBLEMS

Question 1:

$$x^2 + 5x + y^2 - 7y = \frac{251}{2}$$

In the xy-plane, the graph of this equation is a circle. What is the length of the circle's radius?

Question 2:

Consider the circle in the xy-plane defined by the equation

$$(x+2)^2+(y-10)^2=49.$$

Suppose the point (a, b) lies on this circle. Which of the following could be the value of a?

A) -10

B) 3

C) 6

D) 12

Question 3:

$$(x-2)^2+(y-k)^2=49$$

In the xy-plane, the graph of the above equation is a circle. If the point $(9, -2)$ lies on this circle, what is the value of k?

Question 4 (Hint: Believe it or not, you know at least three points on the function and can use a Regression):

A circle in the xy-plane has its center at $(2, -3)$ and has a radius of 7. An equation of this circle is

$$x^2 + y^2 + ax + by + c = 0,$$

where a, b, and c are constants. What is the value of c?

DESMOS CONCEPT #5 CIRCLE EQUATION ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. 12**
- ☐ **2. B**
- ☐ **3. -2**
- ☐ **4. -36**

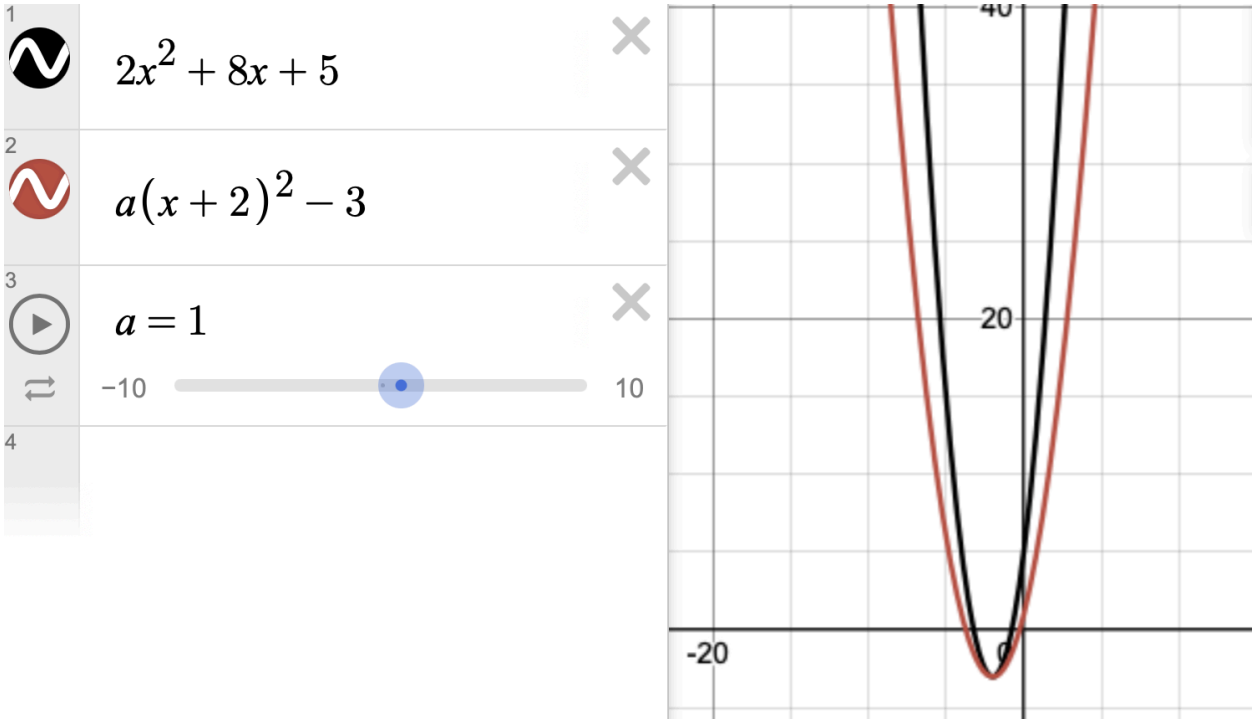
Desmos Concept #6: Equivalent Expressions

WHAT YOU NEED TO KNOW

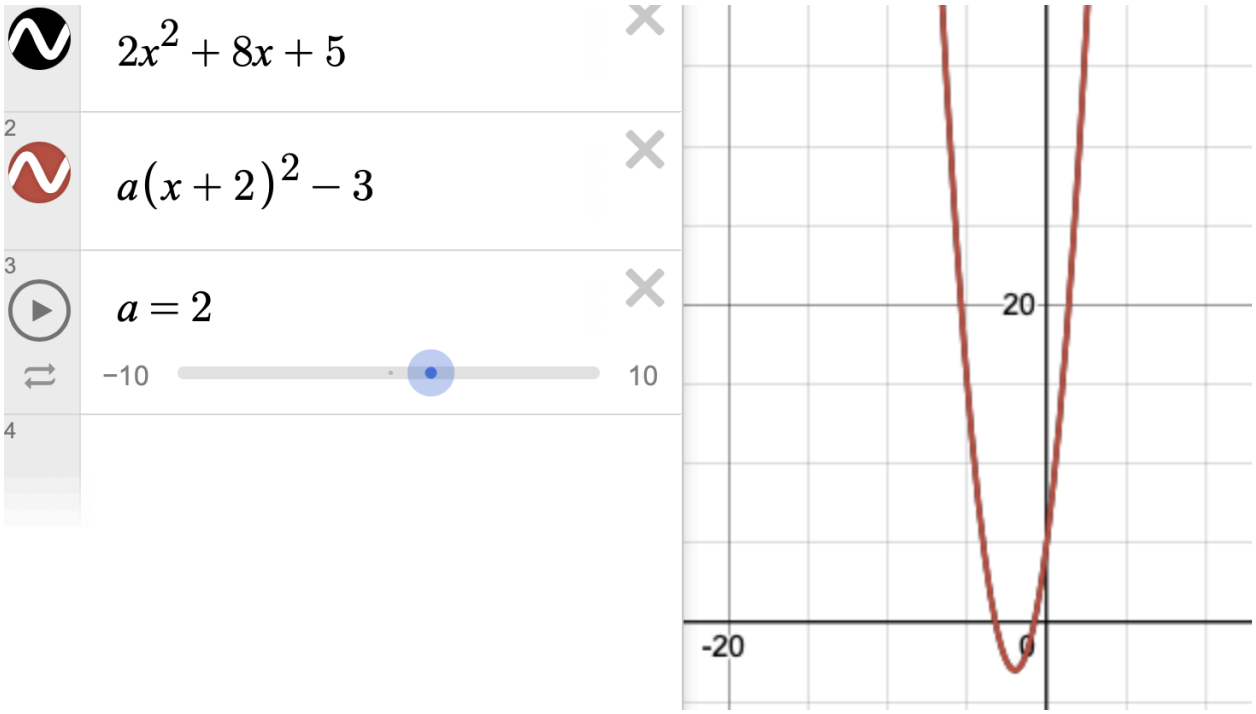
👉 If you see the phrase "**can be rewritten as**," you are looking for an equivalent expression!

💡 **Tip:** Put the two expressions into Desmos and see if the graphs overlap OR if there is a slider, move it until the graphs completely overlap.

Before moving the slider a:



After moving the slider a=2, the red graph completely overlaps the black graph:



DESMOS EQUIVALENT EXPRESSIONS PRACTICE PROBLEMS

Question 1:

The function f is defined by the given equation:

$f(x) = 60(2)^x$

If $g(x) = f(x-4)$, which of the following equations defines the function g?

A) $g(x) = 15(2)^x$

B) $g(x) = \frac{11}{4}(2)^x$

C) $g(x) = \frac{15}{4}(2)^x$

D) $g(x) = 60(6)^x$

Question 2:

The function f is defined by the given equation:

$$f(x) = (2.15)^{\frac{x}{4}}$$

The equation can be rewritten as

$$f(x) = \left(1 + \frac{p}{100}\right)^x,$$

where p is a constant. Which of the following is closest to the value of p?

A) **18**

B) **21**

C) **47**

D) **98**

Question 3:

Which expression is equivalent to

$$\frac{3x(x-5) - 7(x-5)}{4x - 20}, \quad \text{where } x > 5?$$

A. $\frac{x-5}{4}$

B. $\frac{3x-7}{4}$

C. $\frac{3x^2-7x-20}{4x-20}$

D. $\frac{3x^2-22x+35}{4x-20}$

DESMOS CONCEPT #6 EQUIVALENT EXPRESSIONS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. C**
- ☐ **2. B**
- ☐ **3. B**

Desmos Concept #7: Percent Problems as a List

WHAT YOU NEED TO KNOW

if the problem says

A is 370% of B

A is 35% of C

C is p% of B

👉 You can enter these problems in as a list in Desmos, as such:

[A, A, C] ~ [370% of B, 35% of C, p% of B]

PERCENT PROBLEMS AS A LIST PRACTICE PROBLEMS

Question 1:
The mass of object A is 360% of the mass of object B, and the mass of object A is 0.480% of the mass of object C. If the mass of object C is p% of the mass of object B, what is the value of p?

Question 2:
The mass of object A is 275% of the mass of object B, and the mass of object A is 0.550% of the mass of object C. If the mass of object C is pp p% of the mass of object B, what is the value of pp p?

Question 3:
The mass of object A is 150% greater than the mass of object B, and the mass of object A is 60% less than the mass of object C. If the mass of object C is pp p% of the mass of object B, what is the value of pp p?

DESMOS CONCEPT #6 PERCENT PROBLEMS AS A LIST ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. 75,000**
- ☐ **2. 50,000**
- ☐ **3. 625**

When You Should Not Use Desmos

Many problems are easier to solve by hand. Let's look at some use cases when not good to solve with Desmos.

1. Systems with Binomials

Typically, one of the binomial terms easily cancels out and you can solve for the other binomial.

Example:

$$(x-5)-4(y+3)=122$$

$$(x-5)+4(y+3)=434$$

The solution to the given system of equations is (x, y). What is the value of $8(x-5)$?

2. Problems that Need to be Converted to x and y.

Sometimes you can use desmos, you just have to change the variables and make them x and y, or Desmos won't understand it.

Example:

$$6+7r=lm$$

$$7r-5m=5m+11$$

In the given system of equations, l is a constant. If the system has no solution, what is the value of l?

3. Abstract Problems with Inequalities (Draw a Sketch)

For these problems, it is much better to draw a sketch to solve.

Example:

The function f is defined by the given equation, where a and b are integer constants:

$$f(x) = (x - a)(x - b)$$

If $f(13) > 0$, $f(16) < 0$, and $f(19) > 0$, what is one possible value of $a + b$?

4. Problems that Require a Common Denominator

It works better to get a common denominator on problems involving fractions without one.

Example:

Which expression is equivalent to

$$\frac{y+12}{x-8}+\frac{y(x-8)}{x^2y-8xy}\ ?$$

A)

$$\frac{xy+y+4}{x^3y-16x^2y+64xy}$$

B)

$$\frac{xy+9y+12}{x^2y-8xy+x-8}$$

C)

$$\frac{xy^2+13xy-8y}{x^2y-8xy}$$

D)

$$\frac{xy^2+13xy-8y}{x^3y-16x^2y+64xy}$$

5. Quadratics Problems Where You're Only Provided with the Zeros

Although you have two points and the equation of the function, a regression will not work here. If Desmos only has the zeros, it is unable to determine the curve to the parabola. This is where you will need to solve for the ***axis of symmetry.***

Example:

The function f is defined by

$$f(x) = ax^2 + bx + c,$$

where a, b, and c are constants.

The graph of $y = f(x)$ in the xy-plane passes through the points $(10, 0)$ and $(-3, 0)$. If a is an integer greater than 1, which of the following could be the value of $a + b$?

A) 10

B) 7

C) -6

D) -12

6. Product of Solutions

Some problems encourage using a quadratics shortcut. In this case, if they are asking for a product of solutions, use c/a. This is faster than putting it in Desmos with 2 sliders!

Example:

In the given equation, r and s are positive constants.

$$39x^2 + (39s + r)x + rs = 0$$

The product of the solutions to the given equation is krs, where k is a constant. What is the value of k?

7. Multiple Constants (Sliders) and a Complicated Looking Function

You will be moving sliders around in Desmos forever trying to figure this out. If there are multiple constants, there is probably an easier/quicker way by hand.

Example:

The function p is defined by

$$p(x) = a\left((x+7)^2 - b\right)\left((x+7)^2 - c\right),$$

where a, b, and c are constants. In the xy-plane, the graph of $y = p(x)$ passes through the points $(-4, 20)$ and $(2, 276)$.

What is the value of $p(-10) + p(-16)$?

8. Asking for No Solution (Make sure Coefficients are the Same)

This is a problem that would take longer in Desmos for a couple of reasons: (1) the answer is an uncommon fraction, which is hard to obtain in Desmos with a slider and (2) it is just quicker to solve for p by hand, since you know the coefficients in front of x must be the same.

Example:

$$-\frac{3}{19}px + \frac{q}{8} = 10 - \frac{6}{17}x$$

In the given equation, p and q are constants. The equation has no solution. What is the value of p?

9. Circle Problems That Have an Unexpected Constant

$$x^2 + y^2 - 8x + 2y - 16n = 0$$

This is the equation of Circle A.

Circle B has the same center as Circle A but has **twice the diameter** of Circle A and passes through the point $(14, 7)$.

What is the value of n?

WHEN YOU SHOULD NOT USE DESMOS KEY ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. 2224**
- ☐ **2. 10**
- ☐ **3. 31, 32, 33**
- ☐ **4. C**
- ☐ **5. D**
- ☐ **6. 1/39**
- ☐ **7. 296**
- ☐ **8. 38/17**
- ☐ **9. 1.5**

Chapter 3
Algebra

Algebra ❓➕ ❓ 🟰 ❓

Concept #1: Linear Equations

WHAT YOU NEED TO KNOW

1. Equation of a line: _____

m: _____

b:_____

2. Know Perpendicular vs. Parallel Slopes

Example: y = 2x + 7

Perpendicular slope: _____

Parallel slope: _____

3. Know How to Get the Equation of a Line When Given Two Points

Example: The function of a line contains the points (4, -5) and (6, 3).
a. Find the equation by hand

b. Use a linear regression in Desmos to find the equation of the line.

How?

Step 1:

Step 2:

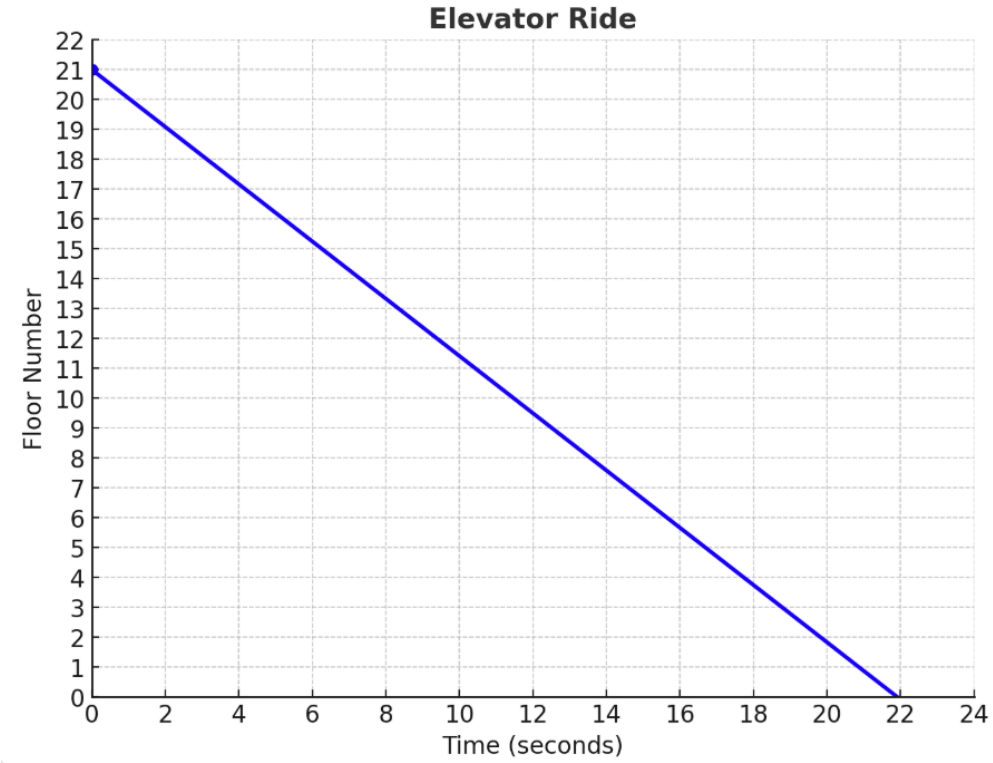
4. Know How to Get the Slope when a line is in Standard Form:
The equation of a line is 5x + 3y = -12. What is the slope of a line perpendicular to this line?

5. Know How to Quickly Get the Slope of The Line from a Table

x	f(x)
3	8
9	23
15	38

Slope: _____
Ok, now how do you solve for the y-intercept?

6. Know How to Quickly Get the Slope of The Line from a Graph



‼️ Note that on SAT problems, they often have the x- and y- axis counting by different amounts

7. Know How to Quickly Get the Slope of The Line in Standard Form

$$Ax + By = C \text{ —-> } \text{Slope} = -A/B$$

What is the slope of 2x - 3y = 12?

Slope = _____

PRACTICE PROBLEMS

Y-intercepts and X-intercepts

Question 1:

In the xy-plane, line k has a slope of 5 and a y-intercept of $(0, -35)$. What is the x-coordinate of the x-intercept of line k?

Question 2:

A line in the xy-plane passes through the points $(6, 2)$, $(0, 5)$, and $(c, 0)$. **What is the value of c?**

Question 3:

A line in the xy-plane passes through the point $(2, 6)$ and crosses the x-axis at the point $(8, 0)$. The line crosses the y-axis at the point $(0, b)$. **What is the value of b?**

Parallel Lines

Question 4:

> Line p is defined by the equation $4x - 7y = 14$. Line q is parallel to line p in the xy-plane. **What is the slope of line q?**

A. -4
B. -7
C. $\frac{4}{7}$
D. $\frac{7}{4}$

Question 5:

"In the xy-plane, line s passes through the point $(0, 0)$ and is parallel to the line represented by the equation

$$y = 50x - 4.$$

If line s also passes through the point $(5, d)$, what is the value of d?"

Perpendicular Lines

Question 6:

Line k is defined by

$$y = 6x + 1.$$

Line j is perpendicular to line k in the xy-plane. What is the slope of line j?

A) -6
B) $-\frac{1}{6}$
C) $\frac{1}{6}$
D) 6

Question 7:

In the xy-plane, line k is defined by

$$x + 2y = 0.$$

Line j is perpendicular to line k, and the y-intercept of line j is $(0, 5)$. Which of the following is an equation of line j?

A) $x + 2y = 5$
B) $y = -2x + 5$
C) $y = 2x + 5$
D) $x - y = 5$

Question 8:

In the xy-plane, line k is given by the equation

$$2x + 5y = 8.$$

Line m is perpendicular to line k and passes through the point $(1, -2)$. Which of the following is an equation of line m?

A) $2x + 5y = 8$
B) $5x - 2y = 9$
C) $5x + 2y = 1$
D) $x - y = -1$

Forming a Linear Equation using Given Points

Question 9:

Line r in the xy-plane has a slope of $-\frac{5}{2}$ and passes through the point $(2, 7)$. Which equation defines line r?

A. $y = -\frac{5}{2}x + 7$
B. $y = \frac{5}{2}x + 2$
C. $y = -\frac{5}{2}x + 12$
D. $y = \frac{2}{5}x - 7$

Question 10:

In the xy-plane, line k passes through the points $(0, -7)$ and $(1, -2)$. Which equation defines line k ?

A) $y = -x + 1$
B) $y = 5x - 7$
C) $y = -2x + 7$
D) $y = \frac{1}{5}x - 7$

Tabular Linear Problems

Question 11:

Consider the linear equation

$$\frac{2}{3}x + \frac{1}{2}y = 6.$$

Which of the following tables lists three ordered pairs (x, y) that all satisfy this equation?

A)

x	y
0	12
9	0
3	8

B)

x	y
0	12
6	12
3	8

C)

x	y
0	12
9	2
3	8

D)

x	y
0	10
6	8
9	0

Question 12:

Consider the linear equation

$$\frac{3}{5}x + \frac{4}{7}y = 12.$$

Which of the following tables lists three ordered pairs (x, y) that all satisfy this equation?

A)

x	y
0	21
5	$\frac{63}{4}$
15	$\frac{21}{4}$

B)

x	y
0	16
5	$\frac{63}{4}$
15	$\frac{21}{4}$

C)

x	y
0	21
5	16
15	$\frac{21}{4}$

D)

x	y
0	21
5	$\frac{63}{4}$
15	5

Question 13:

The table below shows three points on a line in the xy-plane, where k and n are constants:

x	y
2	4
k	7
8	n

If the slope of this line is 3, what is the value of $k + n$?

Question 14:

"The table below shows three values of x and their corresponding values of y. The linear relationship between x and y can be represented by an equation written in the form

$$Ax + By = C,$$

where A, B, and C are constants.

x	y
-1	9
1	13
3	17

What is the value of $\frac{A}{B}$?"

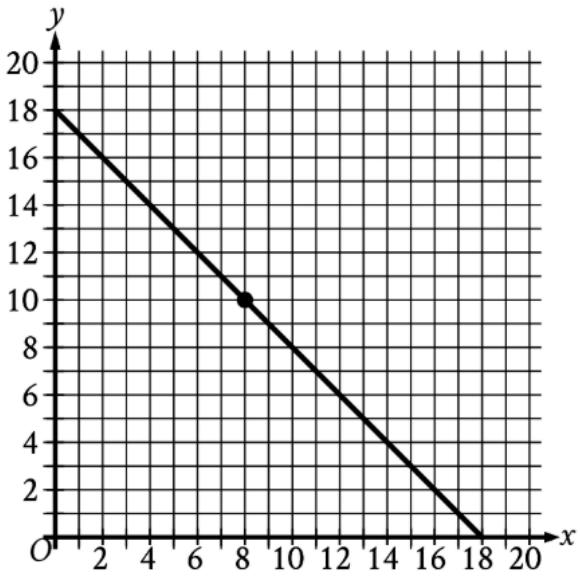
Question 15 (Hint: Run a Linear Auto-Regression):

x	y
-12	-45
6	45

The table shows two values of x and their corresponding values of y. The graph of the linear equation representing this relationship passes through the point $\left(\frac{1}{4}, a\right)$. What is the value of a?

Graphical Linear Problems

Question 16:



The graph in the *xy*-plane models the possible combinations of length x, **in meters (m)**, and width y, in meters, for a rectangle with a perimeter of 36 **m**. Which statement is the best interpretation of the point $(8, 10)$ in this context?

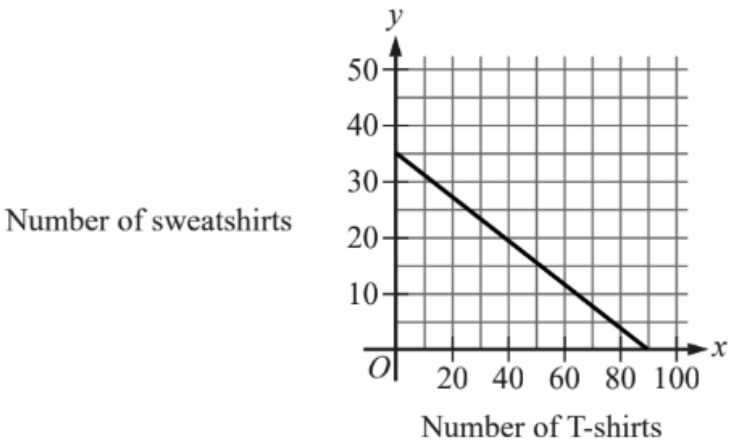
A. The length is 10 **m** less than the perimeter, and the width is 8 **m** less than the perimeter.

B. The length is 10 **m**, and the width is 8 **m**.

C. The length is 8 **m**, and the width is 10 **m**.

D. The length is 8 **m** less than the perimeter, and the width is 10 **m** less than the perimeter.

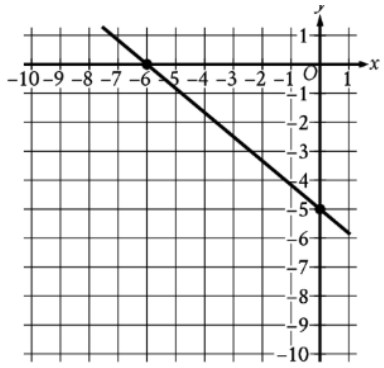
Question 17:



The graph represents the relationship between the number of T-shirts, x, and the number of sweatshirts, y, that Kira can purchase for a school fundraiser. Which equation could represent this relationship?

A. $y = 7x + 18$
B. $7x + 18y = 630$
C. $y = 18x + 7$
D. $18x + 7y = 630$

Question 18:



Line k is shown in the xy-plane. Line j (not shown) is perpendicular to line k. What is the slope of line j?

Linear Regressions

💡 **Tip:** Remember, you can do a regression if you have at least 2 points and know the equation of the function!

Question 19:

The graph of a line in the xy-plane passes through the point $(2, 7)$ and crosses the x-axis at the point $(5, 0)$. The line crosses the y-axis at the point $(0, b)$. What is the value of b?

Question 20:

A linear model estimates the population of a city from 2000 to 2024. The model estimates the population was 60 thousand in 2000, 220 thousand in 2020, and x thousand in 2024. **To the nearest whole number, what is the value of x?**

Question 21:

The table below shows three values of x and their corresponding values of y. The linear relationship between x and y can be represented by an equation of the form

$$Ax + By = C,$$

where A, B, and C are constants.

x	y
0	6
2	0
4	-6

What is the value of $\frac{A}{B}$?

ALGEBRA CONCEPT #1 LINEAR EQUATIONS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. 7**
- ☐ **2. 10**
- ☐ **3. 8**
- ☐ **4. C**
- ☐ **5. 250**
- ☐ **6. B**
- ☐ **7. C**
- ☐ **8. B**
- ☐ **9. C**
- ☐ **10. B**
- ☐ **11. A**
- ☐ **12. A**
- ☐ **13. 25**
- ☐ **14. -2**
- ☐ **15. 16.25 or 65/4**
- ☐ **16. C**
- ☐ **17. B**
- ☐ **18. 1.2 or 6/5**
- ☐ **19. 11.66 or 11.67 or 35/3**
- ☐ **20. 252**
- ☐ **21. 3**

Concept #2: Challenge Linear Problems

⭐When the Y-Intercept Is Different Than What You Expect⭐

👉**Tip:** If a certain amount is being calculated differently than the rest, you need to subtract out that amount from the total so it doesn't double count!

Let's start with a couple of examples so you can get a feel for how to set these up:

Example 1

The cost of renting a backhoe for up to **10** days is **$270** for the first day and **$135** for each additional day. Which of the following equations gives the cost y, in dollars, of renting the backhoe for x days, where x is a positive integer and $x \leq 10$?

A. $y = 270x - 135$

B. $y = 270x + 135$

C. $y = 135x + 270$

D. $y = 135x + 135$

Example 2

For groups of **25** or more people, a museum charges **$21** per person for the first **25** people and **$14** for each additional person. Which function f gives the total charge, in dollars, for a tour group with n people, where $n \geq 25$?

A. $f(n) = 14n + 175$

B. $f(n) = 14n + 525$

C. $f(n) = 35n - 350$

D. $f(n) = 14n + 21$

PRACTICE PROBLEMS

Question 1:

A driving school charges $15 for the first lesson a student takes in a month and $9 for each additional lesson that month. Which equation describes this situation, where y is the total cost (in dollars) of x lessons for one month, and $x > 0$?

A) $y = x + 15$
B) $y = 9x + 15$
C) $y = 9(x - 1) + 15$
D) $y = (x - 1) + 15$

Question 2:

For groups of 30 or more hikers, a guide service charges $25 per person for the first 30 hikers and $16 per person for each additional hiker beyond 30. Which function f gives the total charge (in dollars) for a group of n hikers, where $n \geq 30$?

A) $f(n) = 16n + 270$
B) $f(n) = 25n + 16$
C) $f(n) = 16n + 350$
D) $f(n) = 16n + 750$

Question 3:

The cost of renting a portable generator is $80 for the first day and $30 for each additional day. Which of the following functions $G(d)$ gives the total cost (in dollars) of renting the generator for d days, where d is a positive integer?

A) $G(d) = 30d + 30$
B) $G(d) = 30d + 80$
C) $G(d) = 80d - 30$
D) $G(d) = 50 + 30d$

Question 4:

The cost to rent a film studio from Company X is $1000 for the first 3 hours, plus an additional $60 per hour for each hour after the first 3. If the total cost to rent the studio for t hours (where $t > 3$) is $1300, which equation represents this situation?

A) $1000\,(t - 3) + 60t = 1300$
B) $1000\,(2t) + 60t = 1300$
C) $1000 + 60\,(t - 3) = 1300$
D) $1000 + 60\,(2t) = 1300$

Question 5:

A window repair specialist charges **$220** for the first two hours of repair plus an hourly fee for each additional hour. The total cost for **5** hours of repair is **$400**. Which function f gives the total cost, in dollars, for x hours of repair, where $x \geq 2$?

A. $f(x) = 60x + 100$

B. $f(x) = 60x + 220$

C. $f(x) = 80x$

D. $f(x) = 80x + 220$

Question 6:

A bird-watching group initially observed 200 birds in a certain area and set a goal of reaching 800 birds. According to their model, the population starts at 200 and increases by 30 birds per week for the first two weeks after the observation. After those two weeks, the population increases by 50 birds per week until the goal of 800 birds is reached. At the end of week w (where $w > 2$) after the initial observation, let $p(w)$ be the predicted number of birds still needed to reach the 800-bird goal. Which of the following functions correctly represents $p(w)$?

A) $p(w) = 800 - 50w$
B) $p(w) = 750 + 50w$
C) $p(w) = 640 - 50w$
D) $p(w) = 260 + 50w$

ALGEBRA CONCEPT #2 CHALLENGE LINEAR PROBLEMS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. C**
- ☐ **2. A**
- ☐ **3. D**
- ☐ **4. C**
- ☐ **5. A**
- ☐ **6. C**

Concept #3: Interpreting an Equation

💡**Tips:**
👉 "each" or "per" = coefficient in front of term i.e. '3'
👉 "total" = whole term i.e. '3x'

Let's illustrate with an example:

Sammy sold $100 worth of cookies at her school's bake sale, where c represents the number of chocolate chip cookies sold and p represents the number of peanut butter cookies sold. Here is the equation that represents this scenario:

3c + 2p = 100

What do the following represent:

What represents the cost <u>per</u> chocolate chip cookie? _____

What represents the cost <u>per</u> peanut butter cookie? _____

What represents the <u>total</u> cost of chocolate chip cookies? _____

What represents the <u>total</u> cost of peanut butter cookies? _____

PRACTICE PROBLEMS

Question 1:

A club sold tickets to a carnival to raise funds. Each child ticket sold for $8, and each adult ticket sold for $12. The club earned a total of $576 from selling these tickets. The equation

$$8x + 12y = 576$$

represents this situation. Which of the following is the best interpretation of y in this context?

A) The number of adult tickets sold
B) The number of child tickets sold
C) The price, in dollars, of each adult ticket
D) The price, in dollars, of each child ticket

Question 2:

Matthew is roping off a rectangular corral for his goats. The corral has a length of a feet and a width of b feet. He uses a total of 56 feet of rope to enclose the corral. Which equation represents this situation?

A) $2ab = 56$
B) $a + b = 56$
C) $2a + 2b = 56$
D) $ab = 56$

Question 3:

Hassan put up wire fencing along each edge of a rectangular garden. The garden has a length of x feet and a width of y feet. He used a total of 36 feet of wire fencing.

Which equation represents this situation?

A. $2x + 2y = 36$
B. $x + y = 36$
C. $2xy = 36$
D. $xy = 36$

Question 4:

A professor is creating an exam worth **32 points** in total. The exam will have **1-point** questions and **3-point** questions. Let x be the number of 1-point questions, and y the number of 3-point questions.

Which equation represents this situation?

A) $3xy = 32$
B) $3(x + y) = 32$
C) $x + 2y = 32$
D) $x + 3y = 32$

Question 5:

A group of horticulture students studied the relationship between the length of a plant's largest leaf and the plant's total height. They derived the equation

$$H = 2.32L + 33.5$$

which describes the relationship between the leaf length L, in centimeters, and the plant's estimated height H, in centimeters. Which of the following is the best interpretation of the number 2.32 in this context?

A) The increase in the plant's estimated height, in centimeters, for each 1-cm increase in leaf length
B) The increase in the plant's leaf length, in centimeters, for each 1-cm increase in the plant's estimated height
C) The plant's estimated height, in centimeters, when $L = 0$
D) The difference, in centimeters, between leaf length and plant height

Question 6:

Grace ordered mulch and river rock, which cost a total of $381 for her home. The given equation represents the relationship between the number of cubic yards of mulch, x, and the number of tons of river rock, y, Grace ordered:

$$24.5x + 31.75y = 381$$

How much more, in dollars, did a ton of river rock cost Grace than a cubic yard of mulch?

Question 7:

A group loads money into an account to play arcade games. Each time they play a game, the same amount of money is deducted from the account. The function

$$f(m) = 25 - 4m$$

gives the amount of money (in dollars) left in the account after m games have been played. Which of the following represents the amount of money, in dollars, deducted each time the group plays a game?

A) $4m$
B) 4
C) 25
D) $25 - 4m$

Question 8:

A bakery calculates its monthly profit by subtracting its monthly operating costs from its monthly sales revenue. For a month in which x loaves of bread are produced and sold, the equation

$$12{,}000 = 3.75x - 9{,}200$$

represents the situation. Which statement is the best interpretation of $3.75x$ in this context?

A) The total monthly sales revenue, in dollars, from selling x loaves
B) The monthly sales revenue, in dollars, from each loaf sold
C) The total monthly cost, in dollars, of baking x loaves
D) The monthly cost, in dollars, for each loaf baked

Question 9:

A partially filled container holds 15 milliliters of water and is placed under a faucet that drips 0.05 milliliters of water every 5 seconds. Until the container is full, which of the following can be used to represent the volume v, in milliliters, of water in the container t seconds after it is placed under the faucet, where t is a multiple of 5?

A) $v = 0.01t + 15$
B) $v = 0.02t + 15$
C) $v = 0.05t + 15$
D) $v = 5t$

Question 10:

A factory machine makes three kinds of parts. On a particular day, it makes 8-inch parts, 5-inch parts, and 3-inch parts. The number of 8-inch parts produced is 2 times the number of 5-inch parts, represented by n, and the machine makes 10 of the 3-inch parts. Altogether, the machine produces 70 parts that day. Which equation best represents this situation?

A) $8 \cdot (2n) + 5n + 3 \cdot 10 = 70$
B) $8n + 5n + 3n = 70$
C) $2n + 10 = 70$
D) $3n + 10 = 70$

Question 11:

A manufacturing plant makes 10-inch, 9-inch, and 7-inch frying pans. During a certain day, the number of 10-inch frying pans that the manufacturing plant makes is 4 times the number n of 9-inch frying pans it makes, and the number of 7-inch frying pans it makes is 10. During this day, the manufacturing plant makes 100 frying pans total. Which equation represents this situation?

A. $10(4n) + 9n + 7(10) = 100$
B. $10n + 9n + 7n = 100$
C. $4n + 10 = 100$
D. $5n + 10 = 100$

ALGEBRA CONCEPT #3 INTERPRETING AN EQUATION ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. A**
- ☐ **2. C**
- ☐ **3. A**
- ☐ **4. D**
- ☐ **5. A**
- ☐ **6. 7.25**
- ☐ **7. B**
- ☐ **8. A**
- ☐ **9. A**
- ☐ **10. D**
- ☐ **11. D**

Concept #4: Functions

WHAT YOU NEED TO KNOW

Example: f(2) = 5

2 stands for: _____

5 stands for:_____

Thus, another way to write f(2) = 5 is (____, ____)

Please note that f(x) is a fancy term for _____.

Functions with a Translation

What type of shift is this and in what direction?

f(x) + 3 :_____

f(x - 2): _____

f(x) - 4: _____

f(x + 5): _____

PRACTICE PROBLEMS

Question 1:

The function h is defined by $h(x) = -5x + 35$. What is the value of $h(x)$ when $x = -4$?

Question 2:

The function p is defined by $p(t) = 8t + 5$. What is the value of $p(t)$ when $t = 4$?

Question 3:

If $\frac{1}{4}y + 7 = 10$, what is the value of $4y$?

A) 16

B) 24

C) 36

D) 48

Question 4:

The equation $S = 25t + 200$ gives the total score S that Olivia has after t days of practice. What is Olivia's total score after 8 days of practice?

Question 5:

If $3m + 2 = 17$, what is the value of $9m - 5$?

Question 6:

If $f(x) = x + 5$ and $g(x) = 3x$, what is the value of $5f(2) - g(2)$?

A) -1

B) 14

C) 24

D) 29

Question 7:

The function g is defined by $g(x) = 8x - 200$. If the point $(r, 2r)$ lies on the graph of $y = g(x)$, where r is a constant, what is the value of r?

A) 30

B) 32

C) $\frac{100}{3}$

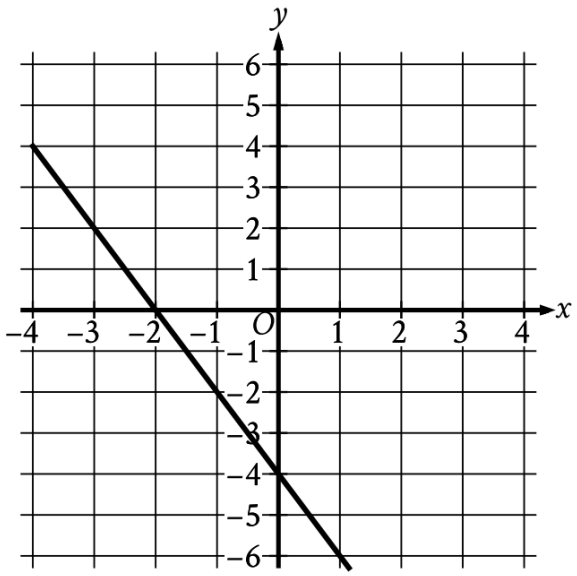
D) 36

Question 8:

The function $G(x) = \frac{9}{5}(x - 273) + 32$ converts a temperature of x kelvins into degrees Fahrenheit. If a temperature increases by 3.50 kelvins, by how many degrees Fahrenheit does the temperature increase?

Question 9:

The graph of $y = f(x) - 11$ is shown.



Which equation defines the linear function f?

A. $f(x) = -13x - 11$

B. $f(x) = -2x + 7$

C. $f(x) = -13x + 7$

D. $f(x) = -2x - 11$

Challenge Problems - Functions that are Non-Linear

Question 10

The function p is defined by

$$p(x) = a\left((x+5)^2 - b\right)\left((x+5)^2 - c\right)$$

where a, b, and c are constants. In the xy-plane, the graph of $y = p(x)$ passes through the points **(-3, 45)** and **(3, 186)**. What is the value of $p(-7) + p(-13)$?

Question 11

The function q is defined by

$$q(x) = a|x-9|^2 - 53|x-9| + b$$

where a and b are constants, and $a > b > 53$. If $q(539) = h$ and $q(-521) = k$, where h and k are constants, what is the value of

$$32(-539)^{h-k} + 347(9)^{k-h}?$$

ALGEBRA CONCEPT #4 FUNCTIONS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. 55**
- ☐ **2. 37**
- ☐ **3. D**
- ☐ **4. 400**
- ☐ **5. 40**
- ☐ **6. D**
- ☐ **7. C**
- ☐ **8. 6.3**
- ☐ **9. B**
- ☐ **10. 231**
- ☐ **11. 379**

Concept #5: Systems of Equations

Systems of Equations are now easier than ever on the Digital SAT.

🔥 **Strategy: Type both equations into Desmos and look for the intersection point.**

PRACTICE PROBLEMS

Question 1:

$$x + 3y = 35$$
$$5x - 17y = -80$$

The system of equations has a solution at (x, y). What is the value of y?

Question 2:

$$y = \frac{x}{4} + 3$$
$$y = -\frac{x}{4} + 46$$

The solution to the given system of equations is (x, y). What is the value of $2y$?

Question 3:

$$\frac{x}{6} + 4(y - 10) = 24$$

$$\frac{x}{2} - 4(y - 10) = 36$$

The solution to the given system of equations is (x, y). What is the value of $12x$?

Question 4:

$$4y = 9x + 10$$

$$-4y = 6x - 22$$

What is the value of $20x$?

Question 5:

$$x + 3y = 17$$

$$2x + y = 5$$

What is the value of $4x + 2y$?

Question 6:

$$\frac{7}{5}x + 3y = 20$$
$$\frac{2}{5}x + 3y = 10$$

The solution to the given system of equations is (x, y). What is the value of $\frac{9}{5}x + 9y$?

Question 7:

$$-x - wy = -275$$
$$3x - wy = 89$$

Given that the graphs intersect at $(q, 12)$, find w.

Question 8:

$$-x - wy = -337$$
$$2x - wy = 47$$

In the given system of equations, w is a constant. In the xy-plane, the graphs of these equations intersect at the point $(q, 19)$, where q is a constant. What is the value of w?

Question 9:

$$18x - 13y = 39$$

$$7x + 12y = 66$$

The solution to the given system of equations is $(x, y) = \left(\frac{p}{47}, \frac{w}{705}\right)$, where p and w are integers. What is the value of p?

Question 10:

In the given system of equations, k is a constant. If the system has no solution, what is the value of k?

$$\frac{2}{3}x + \frac{3}{4}y = 5$$

$$\frac{4}{6}x + \frac{k}{8}y = 1$$

Question 11:

$$5x + 8y = 20$$
$$10x + 16y = 40$$

For each real number r, which of the following points lies on the graph of both equations in the xy-plane?

A. $\left(r,\ -\frac{3}{4}r + 2\right)$

B. $\left(r,\ \frac{2}{5}r + 2\right)$

C. $\left(r + 4,\ -\frac{2}{5}r\right)$

D. $\left(r + 4,\ -\frac{5}{8}r\right)$

Question 12:

$$7x + 6y = 5$$
$$28x + 24y = 20$$

For each real number r, which of the following points lies on the graph of each equation in the xy-plane for the given system?

A. $\left(r,\ -\frac{6r}{7} + \frac{5}{7}\right)$

B. $\left(r,\ \frac{7r}{6} + \frac{5}{6}\right)$

C. $\left(\frac{r}{4} + 5,\ -\frac{r}{4} + 20\right)$

D. $\left(-\frac{6r}{7} + \frac{5}{7},\ r\right)$

ALGEBRA CONCEPT #5: SYSTEMS OF EQUATIONS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

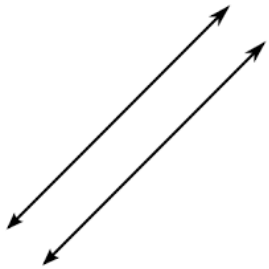
- ☐ **1. 7.96875**
- ☐ **2. 49**
- ☐ **3. 1080**
- ☐ **4. 16**
- ☐ **5. 10**
- ☐ **6. 36**
- ☐ **7. 15.33**
- ☐ **8. 11**
- ☐ **9. 203**
- ☐ **10. 6**
- ☐ **11. D**
- ☐ **12. D**

Concept #6: How Many Solutions?

Here is what you're looking for graphically:

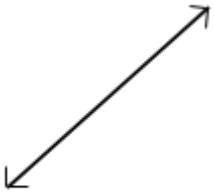
Parallel Lines:

have _____ solution(s)



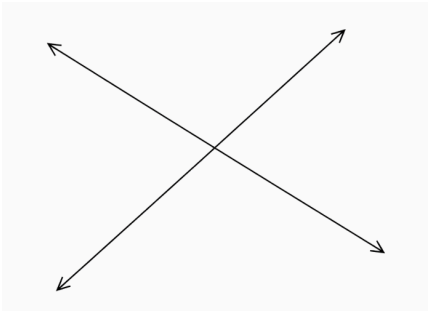
One Line:

have _____ solution(s)



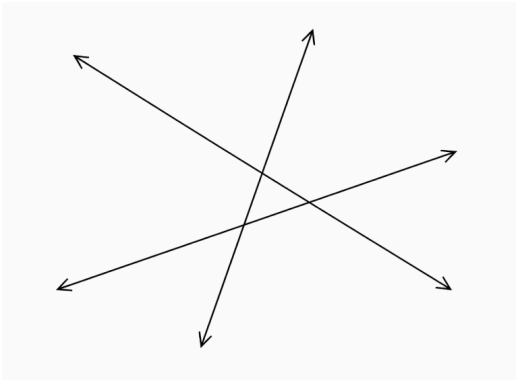
Intersecting Lines:

have _____ solution(s)



System of Three Lines:

This system has_____solution(s)



Algebraic Representations - Fill in the Blanks

	One Equation Example	**Two Equations Example**
No Solution	3x + 7 = _____ - 5	2x + 3y = 20 -2x + _____ = -10
Infinitely Many	3x + 7 = 3x + _____	2x+3y = 20 6x + _____ = 60

💡Tip: Sometimes solving these without Desmos is faster, if you understand what a no solution/infinite solution looks like algebraically.

⭐Time Management Tip - You Can Solve This One in Your Head (Don't Need to Put in Desmos)⭐

What is the value of a for the equation to have no solution?

$$\frac{21}{17}x - 10 = \frac{3}{17}(ax + 12)$$

PRACTICE PROBLEMS

Question 1:

$$16(x-5)=8(2x+7)$$

How many solutions does the equation have?

A. 0

B. 1

C. 2

D. Infinitely many

Question 2:

$$14(x-12)=7(2x-4)$$

How many solutions does the equation have?

A. 0

B. 1

C. 2

D. Infinitely many

Question 3:

$$y = \frac{3}{4}x + 6$$

$$y = cx + 10$$

In the system of equations above, c is a constant. If the system has no solution, what is the value of c?

A. 1
B. $\frac{2}{3}$
C. $\frac{3}{4}$
D. 0

Question 4:

$$y = 4x + 3$$

$$y = ax - 5$$

In the system of equations above, a is a constant. If the system has no solution, what is the value of a?

A. 2
B. 3
C. 4
D. 5

Question 5:

$$y = \frac{2}{7}x + 3$$

One of the two equations in a system of linear equations is given. The system has infinitely many solutions. If the second equation in the system is $y = mx + b$, where m and b are constants, what is the value of b?

A. -3
B. $-\frac{1}{3}$
C. $\frac{1}{3}$
D. 3

Question 6:

$$12(x - m) = 12y + 12m$$

One of the equations in a system of two linear equations is given, where m is a positive constant. The system has no solution.
Which equation could be the second equation in this system?

A) $2x - 3y = 6m$
B) $2x + 3y = 2m$
C) $2x - 2y = 2m$
D) $2x - 3y = 2m$

Question 7:

Which system of linear equations has no solution?

A.

$-2x + 3y = -9$

$2x - 3y = 9$

B.

$2x - 3y = 9$

$3x + 4y = 10$

C.

$2x - 3y = 9$

$-6x + 9y = -27$

D.

$-2x + 3y = 9$

$4x - 6y = 18$

Question 8:

$$\frac{2}{3}x + \frac{1}{4}y = \frac{5}{6}$$

$$ax + y = c$$

If the system of equations has an infinite number of solutions (x, y), what is the value of a?

A. 2

B. $\frac{8}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{4}$

Question 9:

$$-\frac{2}{17}rx + \frac{s}{5} = 9 - \frac{3}{51}x$$

In the given equation, r and s are constants. The equation has no solution.

What is the value of r?

Question 10:

In the given equation, s and r are constants, and $s > 0$.

$$\frac{12x + 28}{4} - \frac{s}{13} = r(x - 8)$$

If the equation has infinitely many solutions, what is the value of s?

ALGEBRA CONCEPT #6: HOW MANY SOLUTIONS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. A**
- ☐ **2. A**
- ☐ **3. C**
- ☐ **4. C**
- ☐ **5. D**
- ☐ **6. C**
- ☐ **7. D**
- ☐ **8. B**
- ☐ **9. ½ OR 0.5**
- ☐ **10. 403**

✨Please note that there is an accompanying Self-Paced Course with video explanations for every problem in this workbook.✨

Go here to learn more:

www.strategictestprep.com/courses

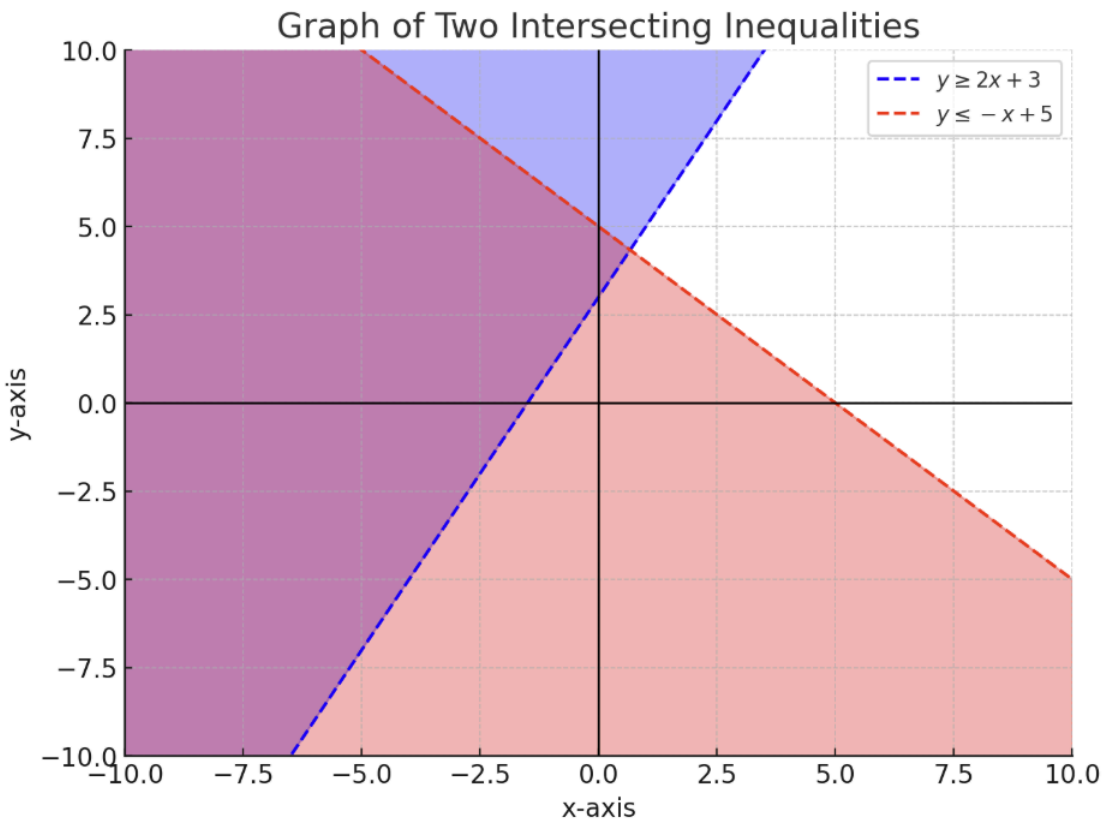
Concept #7 Inequalities

WHAT YOU NEED TO KNOW

Finding the Solution to a System of Inequalities

💡Tip: Type both equations into Desmos.

Example: On the graph below, what color is the solution?.



On this particular graph, is the intersection part of the solution? Why or why not?

Solving an Inequality

💡Tip: When you divide by a negative, you have to _____ the sign!

Example: $-3x \leq 9$

Setting up an Inequality

What inequality sign should you use if the problem says ....

1. "At Most 50" = _____

2. "No more than 12" = _____

3. "At least 220" = _____

4. "A budget of $3400" = _____

5. "More than 300" = _____

PRACTICE PROBLEMS

Question 1:

A garden center classifies a cactus species as tall if its typical height when fully grown is more than 120 centimeters. Which inequality represents the possible heights h, in centimeters, for a tall cactus species?

A) $130 < h < 180$
B) $90 < h < 120$
C) $60 < h < 110$
D) $25 < h < 100$

Question 2:

A circuit has 10 resistors connected in series with a total resistance of 200 ohms. The total resistance of 4 of these resistors is 80 ohms. Which inequality best represents all possible values of the resistance x, in ohms, of one of the other 6 resistors?

A) $0 < x < 10$
B) $0 < x < 120$
C) $120 < x < 200$
D) $80 < x < 200$

Question 3:

A shopper has $30 to spend on pencils and notebooks. Each pencil costs $1.50, and each notebook costs $3.20. Let p be the number of pencils and n the number of notebooks. Which inequality shows how many items the shopper can buy?

A) $1.5p + 3.2n \leq 30$
B) $1.5p + 3.2n \geq 30$
C) $3.2p + 1.5n \leq 30$
D) $3.2p + 1.5n \geq 30$

Question 4:

A dance team aims to raise $1,500 by selling show tickets. They earn $15 per ticket. Which inequality shows the number of tickets t they need to sell?

A) $t + 15 \leq 1,500$
B) $t + 15 \geq 1,500$
C) $15t \leq 1,500$
D) $15t \geq 1,500$

Question 5:

$$y < -5x - 18$$

$$y > -2x - 6$$

Which of the following tables contains only values of x and their corresponding y values that satisfy both inequalities?

A)

x	y
-5	3
-6	9
-8	20

B)

x	y
-5	2
-6	4
-8	-5

C)

x	y
-5	8
-6	9
-8	-5

D)

x	y
-5	5
-6	11
-8	20

Question 6:

$$y > 20$$
$$3x + y < 30$$

If the point $(x, 25)$ is a solution, what could be the value of x?

A) 1

B) 3

C) 5

D) 8

Question 7:

$$y < 35 - 5x$$
$$\frac{y}{5} > 7$$

Which inequality represents the x-values for all solutions (x, y) that satisfy the system?

A) $x > 4$

B) $x < 4$

C) $x > -2$

D) $x < 0$

Question 8:

A freelance graphic designer charges **$25 per hour** for the first **30 hours** worked in a month. For every hour worked **beyond 30 hours**, she charges **$40 per hour**. What is the **minimum number of hours** she must work in a month to earn **at least $1,150**?

A) 39

B) 40

C) 41

D) 42

Question 9:

An employee earns **$20 per hour** for the first **35 hours** worked in a week. For each **additional hour** worked beyond 35, the employee earns **$30 per hour**. What is the **least number of whole hours** the employee must work in a week to earn **at least $980**?

A) 43

B) 44

C) 45

D) 46

Question 10:

A baby elephant weighed 800 pounds at birth and gained 20 pounds per day. If its weight after n days was more than 1,000 pounds but less than 1,200 pounds, what is one possible value of n?

Question 11:

In a set of four consecutive even integers, the first is represented by x. The product of 12 and the third even integer is at most the value of 30 less than the sum of the first and fourth even integers. What is the greatest possible value of x?

Question 12:

In a set of four consecutive odd integers, the smallest is represented by x. The sum of 15 and **twice** the second odd integer is **no more than** the difference between the fourth and the first odd integers **plus 50**. What is the greatest possible value of x?

Question 13:

A small business owner has a budget of **$2,200** to purchase candles. To qualify for discounted pricing, they must buy at least **200** candles. The cost per small candle is **$4.90**, while each large candle costs **$11.60**. What is the maximum number of large candles the owner can purchase while staying within budget and meeting the minimum purchase requirement?

ALGEBRA CONCEPT #7 INEQUALITIES ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. A**
- ☐ **2. B**
- ☐ **3. A**
- ☐ **4. D**
- ☐ **5. D**
- ☐ **6. A**
- ☐ **7. D**
- ☐ **8. B**
- ☐ **9. C**
- ☐ **10. 11, or 12, or 13, or 14, or 15, or 16, or 17, or 18, or 19**
- ☐ **11. -8**
- ☐ **12. 17**
- ☐ **13. 182**

Chapter 4
Advanced Math

Advanced Math 🤓

Concept #1: Quadratics

WHAT YOU NEED TO KNOW

Standard Form (shows the y-intercept): _____

Vertex Form (shows the coordinates of the vertex): _____

Factored Form (shows the zeros/solutions): _____

Quadratic Formula (use when not easily factorable): _____

DISCRIMINANT

$b^2 - 4ac$	**# of solutions**
+	
0	
-	

QUICK SHORTCUTS 🏃‍♂️💨

Sum of the solutions: _____

Product of the solutions: _____

Axis of symmetry: _____

QUADRATICS PRACTICE PROBLEMS

Distributing

Question 1:

If $(x+5)^2 = 80$, what is the value of $x^2 + 10x$?

A) 35

B) 45

C) 55

D) 65

Question 2:

Let $f(x) = x + 2$ and $g(x) = 5x^2 - Mx + 20$.

If

$$f(x) \cdot g(x) = 5x^3 + 40,$$

what is the value of M?

A) 8

B) 9

C) 10

D) 12

Question 3:

$$-7(5x-2)^2+3(5x-1)^2$$

The expression above can be rewritten in the form

$$\frac{r}{4}x^2+\frac{s}{4}x+\frac{t}{4}, \quad \text{where } r, s, \text{ and } t \text{ are constants.}$$

What is $r+s+t$?

A) -80

B) -60

C) -40

D) -20

---

of Real Solutions

Question 4:

How many distinct real solutions does the equation have?

$$(x+5)^2+10(x+5)+25=0$$

A) Exactly one

B) Exactly two

C) Infinitely many

D) Zero

Question 5:

In the equation

$$25x^2 + 50x + c = 0$$

the graph of the parabola has **exactly one** real solution. What is the value of c?

A) 15
B) 20
C) 25
D) 30

Question 6:

If the quadratic equation

$$-x^2 + mx - 4500 = 0$$

has **no real solutions** and m is a positive integer, what is the greatest possible value of m?

A) 132
B) 134
C) 136
D) 138

Question 7:

The quadratic function

$$ax^2 + 40x + c$$

has **at least one real solution**, where a and c are positive integers. What is the greatest possible value of ac?

A) 361

B) 380

C) 400

D) 420

Solutions of a Quadratic

Question 8:

In the given equation, m and n are constants, where $m > 4n$:

$$(x - m)^2 = (m - 4n)(x - m).$$

The sum of the solutions to this equation is $3m + 15$. What is the value of n?

Question 9:

In the given equation, p and q are constants, where $p > 5q$:

$$(x - p)^2 = (p - 5q)(x - p).$$

The sum of the solutions to this equation is $3p + 40$. What is the value of q?

Axis of Symmetry

Question 10:

For the parabola $px^2 + qx + r$, the x-intercepts are $(10, 0)$ and $(m, 0)$.

If $f(25) = f(3)$, what is the value of m?

A) 16

B) 18

C) 20

D) 22

Question 11:

The function $g(z) = pz^2 + qz + r$ has zeros at $z = 1$ and $z = 7$. Suppose p is an integer greater than 1. Which of the following could be the value of $p + q$?

A) -14

B) -16

C) -8

D) -24

Question 12:

When the quadratic function h is graphed in the xy-plane, where $y = h(x)$, its vertex is $(2, -3)$. One of the x-intercepts of this graph is $\left(\frac{11}{4}, 0\right)$. What is the other x-intercept?

A) $\left(\frac{3}{4}, 0\right)$

B) $(-1, 0)$

C) $\left(\frac{5}{4}, 0\right)$

D) $\left(-\frac{5}{4}, 0\right)$

Vertex Form

Question 13:

A quadratic function models the height, in meters, of an object launched from an elevated surface. Measurements show that at $t = 8$ s, the object is 254 m high, and at $t = 12$ s, it is 270 m high.

If the object's height at launch ($t = 0$) was 30 m, what is its height at $t = 17$ s?

A) 140
B) 180
C) 200
D) 220

Question 14:

A quadratic function models the height, in meters, of an object above the ground as a function of time t, in seconds, after launch.

The object is launched from the ground (**0 m**) and reaches its maximum height of **400 m** at $t = 10$ s.

Based on the model, what is the height of the object at $t = 14$ s?

A) 256
B) 288
C) 336
D) 384

Question 15:

A quadratic function h has vertex $(2, -3)$ and passes through the points $(0, 5)$ and $(3, -1)$.

What is $h(4) - h(1)$?

(A) -10

(B) -8

(C) 4

(D) 6

Question 16:

A quadratic function p has vertex $(3, -4)$ and passes through $(2, -6)$. If

$$p(x) = -2(x - 3)^2 - 4,$$

then $a = p(0)$. If

$$T(x) = p(x) - p(0),$$

then what is the positive difference between a and $T(0)$?

Product of Solutions

Question 17:

In the quadratic equation

$$60x^2 + (5p + 8q)x + pq = 0,$$

where p and q are positive constants, the product of the solutions is $b \cdot pq$ for some constant b. What is the value of b?

(A) $\frac{1}{60}$
(B) $\frac{1}{30}$
(C) $\frac{1}{12}$
(D) 1

Question 18:

In the given equation, a and b are positive constants. The product of the solutions to the given equation is kab, where k is a constant. What is the value of k?

$$63x^2 + (63b + a)x + ab = 0$$

Factoring/Expanding a Quadratic

Question 19:

If $R > 0$, $p^2 + q^2 = R$, and $pq = R - 12$, what is $(p + q)^2$ in terms of R?

A) $R - 24$
B) $2R - 12$
C) $2R - 24$
D) $3R - 24$

Question 20:

The equation

$$px^2 + 60x + d$$

has at least one real root and is divisible by the linear factor $rx + s$ (with p, r, s, d all positive integers).

What is the greatest possible value of pd?

A) 820
B) 864
C) 882
D) 900

Question 21:

Consider the expression

$$4x^2 + 22x + d,$$

where **4, 22,** and **d** are positive constants. Suppose $x + 3$ is a factor of this expression **and** the quadratic has real solutions.

What is the greatest possible integer value of d?

A) 28

B) 29

C) 30

D) 31

Question 22:

Which of the following expressions is divisible by $x + 2$?

(A) $4x^2 + 20x + 24$

(B) $4x^2 + 28x + 24$

(C) $4x^2 + 32x + 24$

(D) $4x^2 + 36x + 24$

Quadratic Regression Practice

💡 **Tip:** Remember, you can do a regression if you have at least 2 points and know the equation of the function!

Question 23:

A quadratic function $g(x)$ satisfies $g(0) = 10$, $g(2) = 22$, and $g(6) = 58$. What is the value of $g(8)$?

(A) 70

(B) 78

(C) 82

(D) 90

Getting a Common Denominator

Question 24:

$$\frac{1}{c} - \frac{1}{7} = \frac{1}{3}$$

The equation has exactly one real solution for c. What is that solution?

Draw a Sketch

Question 25:

The function f is defined by the equation

$$f(x) = (x - a)(x - b),$$

where a and b are integer constants. If $f(2) > 0$, $f(5) < 0$, and $f(9) > 0$, what is **one possible value** of $a + b$?

Question 26:

The function g is defined by the equation

$$g(x) = (x - a)(x - b),$$

where a and b are integer constants. If $g(1) > 0$, $g(4) < 0$, and $g(6) > 0$, what is **one possible value** of $a + b$?

Quadratics Answer Key

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. C**
- ☐ **2. C**
- ☐ **3. B**
- ☐ **4. A**
- ☐ **5. C**
- ☐ **6. B**
- ☐ **7. C**
- ☐ **8. -15/4**
- ☐ **9. -8**
- ☐ **10. B**
- ☐ **11. A**
- ☐ **12. C**
- ☐ **13. C**
- ☐ **14. C**
- ☐ **15. D**
- ☐ **16. 22**
- ☐ **17. A**
- ☐ **18. 1/63**
- ☐ **19. D**
- ☐ **20. D**
- ☐ **21. C**
- ☐ **22. A**
- ☐ **23. C**
- ☐ **24. 21/10**
- ☐ **25. 9, 10, 11, or 12**
- ☐ **26. 7 or 8**

Concept #2: Polynomials

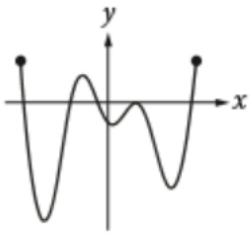
WHAT YOU NEED TO KNOW

Example Polynomial

$$x^2(x-3)(x-1)(x+4) = 0$$

# of distinct roots: _____

Solutions (zeros): x = _____ (double root) x= _____ x= _____ x = _____



How many distinct zeros does this graph have? _____

PRACTICE PROBLEMS

Question 1:

Which expression is equivalent to

$$\left(x^3+5x^2-7x\right) + 4\left(x^2+3\right) ?$$

A) $x^3+5x^2-7x+4x^2+12$

B) $x^3+9x^2-7x+12$

C) x^3+9x^2-7x+4

D) $x^3+5x^2+4x^2-7+12$

Question 2:

$$x^2(x+3)(x-b)=0$$

In the given equation, b is a positive constant. The sum of the solutions of the equation is 5. What is the value of b?

Question 3:

$$y=bx(x-a)(x-a)(x+b)(x-b)$$

In the equation above, a and b are positive constants and $a \neq b$. How many distinct x-intercepts does the graph of the equation in the xy-plane have?

A) Two

B) Three

C) Four

D) Five

Question 4:

$$y = (x - h)^2(x + h)(x + k)$$

The equation above is graphed in the xy-plane. If h and k are positive constants and $h \neq k$, how many distinct x-intercepts does the graph have?

A) 1

B) 2

C) 3

D) 4

Question 5:

$$y = (x - 1)(x + 1)(x + 2)$$

The graph in the xy-plane of the equation above contains the point (a, b). If $-1 \leq a \leq 1$, which of the following is **NOT** a possible value of b?

A) -2

B) -1

C) 0

D) 1

Question 6:

Which of the following is a factor of the polynomial

$$(x^2 - 9)(x + 6)\ ?$$

A) $x + 3$

B) $x - 6$

C) $x + 9$

D) $x - 9$

Question 7:

In the equation

$$y = c\,(x - p)\,(x + q),$$

c, p, and q are positive constants, and $p \neq q$. How many distinct x -intercepts does the graph of this equation have in the xy-plane?

A) 1

B) 2

C) 3

D) 4

Question 8:

In the given equation

$$x^2\,(x+4)\,(x-k)\;=\;0,$$

k is a positive constant. The sum of all solutions of the equation is 6. What is the value of k?

A) 1

B) 2

C) 6

D) 10

Question 9:

Which of the following is a factor of

$$\left(x^2-16\right)\,\left(x+2\right)\,?$$

A) $x-2$

B) $x+4$

C) $x-8$

D) $x+2$

Question 10:

If

$$(x-3)^2(x+1) = 0,$$

how many distinct x-intercepts does this polynomial have in the xy-plane?

A) 1

B) 2

C) 3

D) 4

Question 11:

In the equation

$$(x+2)(x+5)(x-b) = 0,$$

b is a constant. If the **product** of all solutions is -50, what is the value of b?

A) -2

B) -5

C) -10

D) -25

Question 12:

If

$$y = (x - m)(x + n)(x - n),$$

with m and n positive and $m > n$, how many distinct x-intercepts does the graph have?

A) 1

B) 2

C) 3

D) 4

ADVANCED MATH CONCEPT #2 POLYNOMIALS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. B**
- ☐ **2. 8**
- ☐ **3. C**
- ☐ **4. C**
- ☐ **5. D**
- ☐ **6. A**
- ☐ **7. B**
- ☐ **8. D**
- ☐ **9. A**
- ☐ **10. B**
- ☐ **11. B**
- ☐ **12. C**

Concept #3: Exponents

WHAT YOU NEED TO KNOW

Exponent Rules

1. $x^2 * x^5 =$ _____

2. $\frac{x^5}{x^2} =$ _____

3. $x^{-3} =$_____

4. $\sqrt[5]{x^4} =$ _____

5. $x^0 =$ _____

Changing bases to get like bases before solving

Example:

$27^{2x} = 3^{(3x+9)}$ ← (hint: $27 = 3^?$)

When you need to get the y-intercept, set x = _____.

PRACTICE PROBLEMS

Question 1:

Which expression is **equivalent** to $a^{\frac{7}{8}}$ for $a > 0$?

A) $\left(a^7\right)^{\frac{1}{8}}$

B) $\left(a^8\right)^{\frac{1}{7}}$

C) $a^{\frac{8}{7}}$

D) $\left(a^7\right)^{\frac{1}{9}}$

Question 2:

Which expression is **equivalent** to $a^{\frac{11}{12}}$ for $a > 0$?

A) $\sqrt[12]{a^{11}}$

B) $a^{\frac{12}{11}}$

C) $\sqrt[11]{a^{12}}$

D) $a^{\frac{11}{13}}$

Question 3:

If $f(x) = 5^{2x}$, which of the following is an equivalent expression?

A) $\left(5^2\right)^x$

B) $5^{\frac{2}{x}}$

C) $\sqrt{5^{2x}}$

D) 5^{2+x}

Question 4:

If $2^{x+1} = 16$, what is the value of x?

A) 2

B) 3

C) 4

D) 5

Question 5:

Two positive numbers a and b satisfy $\sqrt[3]{a} = \sqrt{b}$. For what value of x is

$$a^{6x-1} = b^2 \text{ ?}$$

A) $\frac{1}{4}$

B) $\frac{3}{8}$

C) $\frac{7}{18}$

D) $\frac{4}{9}$

Question 6:

A function h is defined by $h(x) = c\,(4^x)$. If $h(3) = 192$, what is the value of $h(5)$?

A) 256

B) 768

C) 3072

D) 4096

Question 7:

Suppose

$$\sqrt[4]{p^2} = \sqrt{t} \quad \text{and} \quad t = p^{3n-2},$$

for constants $p > 1$ and $t > 1$. What is the value of n?

A) 1

B) 2

C) 3

D) 4

Question 8:

Which expression is **equivalent** to

$$h(x) = \frac{3^{2x+1}}{9^{x-2}} \quad \text{for } x > 0\,?$$

(Note that $9 = 3^2$.)

A) 3^5

B) 3^{2x-1}

C) 3^2

D) 3^{2x+5}

Question 9:

Which of the following is an equivalent form of

$$\frac{4^{3x-1}}{16^{x-2}} \quad \text{for } x > 0\,?$$

(Note: $16 = 4^2$.)

A) 4^{-1}

B) 4^{3x+3}

C) 4^{x+3}

D) 4^{x-3}

Question 10:

For the function

$$g(x) = 7 \cdot 2^x - 5,$$

which of the following is the **y**-intercept?

A) -5

B) 2

C) 9

D) 7

Question 11:

A function G is defined by

$$G(x) = 8^x + m.$$

The graph of G passes through $(0, -5)$.

What is the value of m?

A) -13

B) -8

C) -7

D) -6

ADVANCED MATH CONCEPT #3 EXPONENTS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. A**
- ☐ **2. A**
- ☐ **3. A**
- ☐ **4. B**
- ☐ **5. C**
- ☐ **6. C**
- ☐ **7. A**
- ☐ **8. A**
- ☐ **9. C**
- ☐ **10. B**
- ☐ **11. D**

Concept #4: Exponential Functions

WHAT YOU NEED TO KNOW

An exponential function has a **variable** in the **exponent**.

Example Equation:

Example Graph (Draw picture):

It is important to know the difference between a linear and exponential function.

Circle the words that stand for **exponential functions.**

Constant	**Rate of Change**	**Population**
Doubles	**Triples**	**Slope**

To get the y-intercept of an exponential function, you make x = _____.

$5^0 =$ ____

Know the exponential growth/decay equation: $P(1 \pm r)^t$

P:_____

r:_____

t:_____

Example 1: $500(1.03)^{12}$

Is this a growth or decay? _____

What is the growth rate? _____

How much time has transpired? _____

What was your initial amount? _____

Example 2: $750(.87)^{\frac{7}{12}}$

Is this a growth or decay? _____

What is the growth rate? _____

How much time has transpired? _____

What was your initial amount? _____

GROWTH/DECAY PRACTICE PROBLEMS

Question 1:

At the start of an experiment, the temperature of a solution is 25 °C. The temperature rises by a factor of 1.5 every 10 minutes. Which equation best represents y, the temperature (in °C) after t minutes?

A) $y = 25 \cdot (1.5)^{t}$
B) $y = 25 \cdot (1.5)^{\frac{t}{10}}$
C) $y = 25 + (1.5)^{\frac{t}{10}}$
D) $y = 25 \cdot (10)^{1.5t}$

Question 2:

In a certain region, a population of trout is monitored each year from 2015 to 2025. On average, the population **decreases** by 5% annually (meaning each new year it is 95% of the previous year's count). If there were 920 such trout in 2015, which expression represents the population in 2025?

A) $920 \cdot (0.95)^{5}$
B) $920 \cdot (0.05)^{10}$
C) $920 \cdot (1.05)^{10}$
D) $920 \cdot (0.95)^{10}$

Question 3:

The function $f(t) = 60{,}000 \cdot 2^{\frac{t}{410}}$ gives the number of bacteria in a population t minutes after an initial observation. How much time, in minutes, does it take for the number of bacteria in the population to double?

Question 4:

A certain colony of bacteria is modeled by

$$B(x) = 800\left(0.85\right)^{\frac{x}{4}},$$

where x is measured in hours since the start of observation. According to the model, how many bacteria are present at $x = 0$?

A) 0

B) 85

C) 800

D) 940

Question 5:

A bank account's balance, in dollars, is modeled by

$$f(t) = 12{,}000\,(1.06)^{4t},$$

where t is measured in years since the account was opened. Which statement is the **best interpretation** of the factor $(1.06)^{4t}$?

A) Every quarter year, the balance increases by 6% of the previous quarter's balance.

B) Every year, the balance increases by 6% of the previous year's balance.

C) Every 4 years, the balance is multiplied by 1.06.

D) Every 6 years, the balance increases by $12{,}000 \times 0.06$.

Question 6:

A certain population $P(t)$, measured in thousands, satisfies

$$P(t) = 400(1.04)^{t},$$

where t is the number of years after 2010.

According to this model, which statement is true?

A) The population **increases** by 4% each year, starting at 400 thousand in 2010.

B) The population **decreases** by 96% each year, starting at 400 in 2010.

C) The population **increases** by 1.04 persons each year, starting at 400 in 2010.

D) The population is 400 thousand every 4 years.

Question 7:

The function F is defined by

$$F(t) = 600(0.90)^t,$$

where t is the number of years since 2015 and $F(t)$ is measured in liters. Which is the best interpretation of this model?

A) The volume starts at 0.90 liters in 2015 and increases by 600 liters each year.

B) The volume starts at 600 liters in 2015 and decreases by 10% each year.

C) The volume starts at 600 liters in 2015 and increases by 90% each year.

D) The volume starts at 0 liters in 2015 and increases by 600% each year.

Question 8:

A town's population M (in thousands) is modeled by

$$M(t) = 500\,(1.01)^{4t},$$

where t is the number of years since 2020. According to the model, the town's population **increases** by $n\%$ every 4 years. What is n (rounded to the nearest tenth of a percent)?

A) 1.0

B) 4.0

C) 4.1

D) 10.0

Question 9:

The function $P(t) = 290(1.04)^{(\frac{4}{6})t}$ models the population, in thousands, of a certain city t years after 2005. According to the model, the population is predicted to increase by $n\%$ every 18 months. What is the value of n?

OTHER EXPONENTIAL FUNCTION PRACTICE PROBLEMS

Question 10:

The function

$$g(x) = 2^x - 5$$

passes through the point $(3, 3)$. If $g(k) = -1$, what is the value of k?

A) 1

B) 2

C) 4

D) 6

Question 11:

The function f is defined by

$$f(x) = a^x + b.$$

Its graph passes through the points $(0, 5)$ and $(1, 10)$. Which is a possible value of b?

A) 3

B) 4

C) 5

D) 9

Question 12:

A quantity Q starts at 50 and **doubles every 6 days**. Which of the following models Q after d days?

A) $Q(d) = 50 + 2d$

B) $Q(d) = 50 \cdot (6)^d$

C) $Q(d) = 50 \cdot 2^{\frac{d}{6}}$

D) $Q(d) = 50 \cdot \left(\frac{1}{2}\right)^d$

Question 13:

Consider the exponential function $f(x) = 6^x$. Let $f(2) = k$. Which of the following **rewrites** of $f(x)$ shows k as a **coefficient**?

A) $f(x) = \left(6^2\right)\left(6^x\right)$
B) $f(x) = 36\left(6\right)^{x-2}$
C) $f(x) = \left(36\right)^x$
D) $f(x) = 6^{x-2}$

Question 14:

A certain population is modeled by $P(t) = A\left(1 + r\right)^t$, where t is measured in years. If $P(0) = 500$ and $P(2) = 605$, what is $P(5)$ (rounded to the nearest integer)?

A) 720
B) 805
C) 818
D) 890

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

CONCEPT #4: Exponential Growth/Decay Functions ANSWER KEY

- ☐ 1. B
- ☐ 2. D
- ☐ 3. 410
- ☐ 4. C
- ☐ 5. A
- ☐ 6. A
- ☐ 7. B
- ☐ 8. A
- ☐ 9. 4

CONCEPT #4: Other Exponential Functions ANSWER KEY

- ☐ 10. B
- ☐ 11. B
- ☐ 12. C
- ☐ 13. B
- ☐ 14. B

Concept #5: Absolute Value Expressions

WHAT YOU NEED TO KNOW

$|5| =$_____

$|-5| =$_____

Thus, if we want to find x for $|2x + 5| = 10$, we know either

$2x + 5 =$ _____ or $2x + 5 =$ _____

The graph of an absolute value function, $y = |x|$, looks like this:

Notice once you've drawn the graph that for most points, there are two x's to every y.

PRACTICE PROBLEMS

Question 1:

Solve for the **positive** value of x:

$$\left|-4x-6\right| = 14.$$

A) -5

B) 2

C) 3

D) 5

Question 2:

If

$$\left|x-7\right| + 80 = 100,$$

what is the **sum** of the solutions to the equation?

A) -13

B) 14

C) 20

D) 27

Question 3:

Define $f(x) = \left| x - 8 \right|$.

If $f(3) - f(a) = -30$, which of the following is the **positive** solution for a?

A) -27

B) 35

C) 42

D) 43

Question 4:

Let $f(x) = \left| 65 - 3x \right|$. If $f(k) = 2k$, which of the following is the **smaller** solution for k?

A) 13

B) 26

C) 39

D) 65

Question 5:

In the equation

$$20\left|x-5\right| = m,$$

there is **exactly one** solution for x if and only if which of the following is true about m?

A) $m = 0$
B) $m = 20$
C) $m = -5$
D) $m = 5$

Question 6:

Which of the following absolute-value equations has **no real solutions**?

A) $\left|2x+7\right| = 3$
B) $\left|x-1\right| = -4$
C) $\left|-3x+2\right| = 2$
D) $3\left|x\right| = 0$

Question 7:

If $f(x) = \left| 2x - 1 \right|$ and $f(x) = 7$, what is the **sum** of the solutions for x?

A) 1

B) 4

C) -3

D) -4

Question 8:

Let $g(x) = \left| x - 2 \right| + 1$. Which of the following ordered pairs **lies on** the graph of g?

A) $(2, 2)$

B) $(3, 2)$

C) $(4, 2)$

D) $(3, 3)$

Question 9:

Which set of real x satisfies

$$\left| 3x - 4 \right| < 5 \text{ ?}$$

A) $x < -\frac{1}{3}$

B) $-\frac{1}{3} < x < 3$

C) $x > 3$

D) $-3 < x < \frac{1}{3}$

Question 10:

What is the **minimum** value of the function

$$y = |x + 2| - 3?$$

A) -5

B) -3

C) 0

D) 2

Question 11:

If $f(x) = |4x - 5|$ and $2f(1) = f(x)$, what is the **sum** of the solutions for x?

A) 2

B) 2.25

C) 2.5

D) 3

Question 12:

What is the **positive difference** between the two solutions of

$$|x-6| = |2x+1|?$$

A) 8

B) 8.3

C) $\frac{26}{3}$

D) 9

Question 13:

How many solutions does the equation $|x+4| = -8$ have?

A) 0

B) Exactly one

C) Exactly two

D) More than two

ADVANCED MATH CONCEPT #5: ABSOLUTE VALUE EXPRESSIONS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. B**
- ☐ **2. B**
- ☐ **3. D**
- ☐ **4. A**
- ☐ **5. A**
- ☐ **6. B**
- ☐ **7. A**
- ☐ **8. B**
- ☐ **9. B**
- ☐ **10. B**
- ☐ **11. C**
- ☐ **12. C**
- ☐ **13. A**

Concept #6: Circle Equation

WHAT YOU NEED TO KNOW

1. A circle equation is in the form
$(x - h)^2 + (y - k)^2 = r^2$, *where* (h, k) *is the center and* r *is the radius.*

Example: $(x - 2)^2 + (y + 5)^2 = 16$

center: (____, ____)

radius:

2. If they don't give it to you in that form, use DESMOS!

3. You may on occasion need the midpoint formula (to find the center) or the distance formula (to find the length of the radius).

Midpoint formula = _____

Distance formula = _____

PRACTICE PROBLEMS

Question 1:

What is the center of the circle defined by

$$(x-2)^2 + (y+9)^2 = 1\ ?$$

A) $(2, -9)$
B) $(2, 9)$
C) $(-2, -9)$
D) $(-2, 9)$

Question 2:

A circle in the xy-plane is defined by

$$(x+5)^2 + (y-6)^2 = 81.$$

If the point (a, b) lies on this circle, which of the following **could** be the x-coordinate a?

A) -14
B) -10
C) 6
D) 10

Question 3:

Which of the following points lies on the circle

$$(x+1)^2 + (y-4)^2 = 9\,?$$

A) $(2, 4)$
B) $(3, 5)$
C) $(-1, 4)$
D) $(2, 5)$

Question 4:

Which of the following points lies on the circle

$$(x+2)^2 + (y-1)^2 = 16\,?$$

A) $(2, 1)$
B) $(-2, 1)$
C) $(2, 3)$
D) $(6, 1)$

Question 5:

A circle in the xy-plane has center $(4, -8)$ and radius 10.

An equation of this circle can be written in the form

$$x^2 + y^2 + Ax + By + C = 0,$$

where A, B, and C are constants. What is the value of C?

A) -20

B) -80

C) -120

D) 20

Question 6:

Which of the following points lies on the circle

$$(x + 4)^2 + (y - 10)^2 = 25?$$

A) $(-4, 6)$

B) $(-4, 15)$

C) $(0, 10)$

D) $(-2, 12)$

Question 7:

A circle in the plane has center $(2, -3)$ and radius 15.

Which is the **standard form** of its equation?

A) $(x-2)^2+(y+3)^2=225$
B) $(x+2)^2+(y-3)^2=225$
C) $(x+2)^2+(y-3)^2=15^2+2$
D) $(x-2)^2+(y-3)^2=15^2$

Question 8:

The circle given by

$$(x+1)^2+(y+5)^2 = 400$$

has center $(-1, -5)$ and radius 20.

Which of the following points **lies on** this circle?

A) $(0, 15)$
B) $(19, 5)$
C) $(19, -5)$
D) $(0, -5)$

Question 9:

Consider the circle defined by

$$x^2 + y^2 + 4x - 8y + 16 = 0.$$

What is the length of this circle's radius?

A) 2

B) 3

C) 4

D) 6

Question 10:

The equation

$$x^2 + y^2 + 2x + 6y = -9$$

describes a circle in the xy-plane. What is the length of its radius?

A) 0

B) 1

C) 2

D) 3

Question 11:

Which of the following points **does not** lie on the circle

$$(x-2)^2+(y+1)^2 = 25\,?$$

A) $(2, -6)$
B) $(7, -1)$
C) $(2, 4)$
D) $(7, -2)$

Question 12:

In the xy-plane, the circle

$$x^2 + y^2 + 3x - 2y = 12$$

has center (h, k). What is $h + k$?

A) -2
B) -1
C) $-\frac{1}{2}$
D) $\frac{1}{2}$

ADVANCED MATH CONCEPT #6: CIRCLE EQUATION ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. A**
- ☐ **2. A**
- ☐ **3. A**
- ☐ **4. A**
- ☐ **5. A**
- ☐ **6. B**
- ☐ **7. A**
- ☐ **8. C**
- ☐ **9. A**
- ☐ **10. B**
- ☐ **11. D**
- ☐ **12. C**

Concept #7: Isolating a Variable (aka Equivalent Expressions)

💡 **Tip:** Please be sure to write out all of your steps on paper, otherwise it is easy to make a careless mistake!

💡 **Tip:** If you are trying to isolate a variable and it is in multiple terms, get all of those terms to one side and then **factor out** the variable you need.

Example: Isolating q
q-p**q**+r**q** = 5s
q(1-p+r) = 5s

At this point, you can just divide both sides by 1-p+r and then you're done!

PRACTICE PROBLEMS

Question 1:

A certain capsize screening value C for a sailboat is given by

$$C = \frac{6B}{\sqrt[4]{D}},$$

where B is the beam (in feet) and D is the displacement (in pounds).

Which of the following equations correctly expresses D in terms of C and B?

A) $D = \frac{(6B)^4}{C}$

B) $D = \frac{C^4}{6B}$

C) $D = \left(\frac{6B}{C}\right)^4$

D) $D = \left(\frac{C}{6B}\right)^4$

Question 2:

An equation relates j, p, and c by

$$j = \frac{p}{c} \times 72.$$

Which choice **correctly** expresses c in terms of j and p?

A) $c = \frac{p}{72j}$

B) $c = \frac{72p}{j}$

C) $c = \frac{j}{72p}$

D) $c = \frac{pj}{72}$

Question 3:

The quantity L is given by

$$L = \frac{100}{m} + 40n,$$

where m and n are variables.

Which equation correctly expresses n in terms of L and m?

A) $n = \frac{L - 100}{40m}$

B) $n = \frac{100 + L}{40m}$

C) $n = \frac{L - \frac{100}{m}}{40}$

D) $n = \frac{mL - 100}{40}$

Question 4:

An English teacher uses the formula

$$E = 0.9K + 1.8F$$

to calculate a composite score E, where K is the number of correct multiple-choice answers and F is the number of free-response points.

Which of the following equations correctly expresses K in terms of E and F?

A) $K = \dfrac{E - 1.8F}{0.9}$

B) $K = \dfrac{E}{0.9} - 1.8F$

C) $K = 0.9\,E - \dfrac{1.8F}{0.9}$

D) $K = \dfrac{1.8F - E}{0.9}$

Question 5:

The equation

$$b^2 + 7c = 12\,d$$

relates real numbers b, c, and d.

Which choice expresses b **in terms of** c and d?

A) $b = \sqrt{12\,d - 7\,c}$

B) $b = \dfrac{12\,d - 7\,c}{b}$

C) $b = 12\,d - 7\,c$

D) $b = \dfrac{7\,c - 12\,d}{b}$

Question 6:

An anthropologist uses the formula

$$h = 3.5\ell + 90$$

to estimate the height h (in centimeters) of a person given ulna length ℓ (in centimeters).

Which of the following equations correctly gives ℓ in terms of h?

A) $\ell = \dfrac{h - 90}{3.5}$

B) $\ell = \dfrac{3.5h - 90}{h}$

C) $\ell = 3.5h - 90$

D) $\ell = \dfrac{90 - h}{3.5}$

Question 7:

In electronics, the equivalent capacitance C of two capacitors in parallel is

$$C = d + e,$$

but a certain arrangement modifies this to

$$\frac{1}{C} = \frac{1}{d} + \frac{1}{e} + \frac{1}{f}.$$

Which of the following expresses C in terms of d, e, and f?

A) $C = d + e + f$

B) $C = \dfrac{def}{d + e + f}$

C) $C = \dfrac{1}{def}\left(\dfrac{1}{d} + \dfrac{1}{e} + \dfrac{1}{f}\right)^{-1}$

D) $C = \dfrac{def}{de + df + ef}$

Question 8:

The equation

$$y = \frac{x + w}{z}$$

relates positive variables w, x, and z.

Which choice **correctly** expresses x in terms of w, y, and z?

A) $x = yz - w$

B) $x = y + \frac{w}{z}$

C) $x = \frac{y}{wz}$

D) $x = \frac{w - yz}{y}$

Question 9:

In the equation

$$\frac{1}{7b} = \frac{11x}{y},$$

the variables b, x, and y are positive.

Which equation **correctly** expresses x in terms of b and y?

A) $x = \frac{y}{77b}$

B) $x = \frac{11y}{7b}$

C) $x = \frac{7b}{11y}$

D) $x = 77\frac{y}{b}$

Question 10:

A quantity A is given by the equation

$$A = P(t + 2),$$

where $P \neq 0$.

Which of the following gives t in terms of A and P?

A) $t = \dfrac{A}{P} - 2$

B) $t = \dfrac{2A - P}{P}$

C) $t = \dfrac{P}{A} - 2$

D) $t = \dfrac{A - 2}{P}$

Question 11:

A seller's rating R is computed by

$$R = \frac{F}{N + F},$$

where F is the number of **favorable** reviews and N is the number of **unfavorable** reviews.

Which equation correctly expresses F in terms of R and N?

A) $F = \dfrac{RN}{1 - R}$

B) $F = \dfrac{N}{R} - 1$

C) $F = N(1 - R)$

D) $F = \dfrac{N - R}{N}$

Question 12:

The given equation relates the positive variables p, q, r, and s:

$$\frac{63}{p} = \frac{63}{q} - \frac{63}{r} - \frac{63}{s}$$

Which of the following is equivalent to q in terms of p, r, and s?

A) $q = p + r + s$

B) $q = \frac{prs}{pr+ps+rs}$

C) $q = prs$

D) $q = \frac{63}{p+r+s}$

ADVANCED MATH Concept #7: Isolating a Variable ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. C**
- ☐ **2. B**
- ☐ **3. C**
- ☐ **4. A**
- ☐ **5. A**
- ☐ **6. A**
- ☐ **7. D**
- ☐ **8. A**
- ☐ **9. A**
- ☐ **10. A**
- ☐ **11. A**
- ☐ **12. B**

Chapter 5
Problem Solving & Data Analysis

Problem-Solving and Data Analysis

Concept #1: Percents

WHAT YOU NEED TO KNOW FOR PERCENTS

How to write a percent in different forms (decimal and fraction)

Examples:

35% = _____ = _____

253% = _____ = _____

1.57% = _____ = _____

0.02% = _____ = _____

How to set up a percents equation to solve

Example 1:

90 is what percent of 250?

Equation: _____

Example 2:

250% of x is 20. What is x?

Equation: _____

Example 3:

A quantity x is increased 400% and the result is 60. What is the value of x?

(Note: why is the answer 12?)

Percent Change Formula:

💡 **Tip:** If the SAT problem says p is a percent and they ask for p, do NOT give them a decimal. You will need to enter the answer in percent form.

Example:

You solve for p and get p=0.375

You will enter _____ as your answer.

PRACTICE PROBLEMS

Single Step Percentage Calculations

Question 1:

Julia purchased 900 feet of fencing. She used 75% of this fencing to surround a vegetable garden. How many feet of fencing did Julia use to surround the vegetable garden?

A) 15
B) 225
C) 675
D) 860

Question 2:

Levi and Marissa each have stamp collections. The number of stamps in Levi's collection is 250% of the number of stamps in Marissa's collection. If there are 600 stamps in Marissa's collection, how many stamps are in Levi's collection?

A) 150
B) 600
C) 1200
D) 1500

Question 3:

A scientist analyzed a soil sample with a mass of 1,000 grams and determined that it contained 220 grams of water. What is the percentage of water, by mass, in this soil sample?

A) 10%
B) 18.9%
C) 22%
D) 24%

Algebraic Percentage Relationship

Question 4:

If $a > 0$ and a is 62% of b, which expression represents b in terms of a?

A) $\frac{100}{62}a$
B) $\frac{62}{100}a$
C) $\frac{100}{38}a$
D) $\frac{38}{100}a$

Question 5:

For $x > 0$, the function f is defined as follows:
$f(x)$ equals **175%** of x.

Which of the following could describe this function?

A) Decreasing exponential
B) Decreasing linear
C) Increasing exponential
D) Increasing linear

Consecutive Percentage Applications

Question 6:

A real estate company offers a series of three webinars. 5,000 people attended the first webinar. 40% of the people who attended the first webinar attended the second webinar, and 25% of the people who attended the first and second webinars attended the third webinar. How many people attended all three webinars?

Question 7:

The value of a collectible comic book increased by 130% from the end of 2015 to the end of 2016 and then decreased by 15% from the end of 2016 to the end of 2017. What was the net percentage increase in the value of the comic book from the end of 2015 to the end of 2017?

A) 85.0%
B) 95.5%
C) 110.5%
D) 130.0%

Question 8:

At the start of a week, there was an equal number of two mite species: a predator and its prey. At the end of the week, the number of prey had increased by 1,500% of the number of prey at the start, and the number of predators had increased by 100% of the number of predators at the start. The number of prey at the end of the week was $p\%$ greater than the number of predators at the end of the week. What is p?

Multi-Group/Multi-Step Percent Calculations

Question 9:

The composition of an animal is defined as the muscles, bones, and fat of the animal. A scientist studied a large goat and determined that the goat had **84 kilograms of muscle**, which made up approximately **60.5%** of its composition. Of the remaining composition of this goat, approximately **40.0%** was bone, and the remainder was fat. Based on these approximations, to the nearest tenth, how many kilograms of this goat's composition was bone?

Question 10:

On a plot of land, **60.0%** of the square footage is farmland and the remaining square footage is pasture. There are buildings on exactly **20.0%** of the farmland, and there are buildings on exactly **10.0%** of the pasture. If there are buildings on exactly $p\%$ of the square footage of the plot of land, what is the value of p?

Question 11:

While the mass of an object is the same everywhere, the weight of an object is not the same on different planets. An object has a weight of **180.00 pounds** on Earth and a weight of **220.00 pounds** on Neptune. The object's weight on Jupiter is **310%** of its weight on Earth. If the object's weight on Neptune is $x\%$ of its weight on Jupiter, which of the following is closest to the value of x?

A) 25.1
B) 39.4
C) 45.9
D) 75.1

Question 12:

A researcher surveyed undergraduate students, graduate students, and postdoctoral students. The number of undergraduate students surveyed was **2,500%** of the number of postdoctoral students surveyed, and the number of graduate students surveyed was **40%** of the number of undergraduate students surveyed. If there were **2,000** graduate students surveyed, what was the sum of the number of undergraduate students and postdoctoral students surveyed?

Advanced Percentage Relationships

Question 13:

The mass of object A is **350%** of the mass of object B, and the mass of object A is **0.08%** of the mass of object C. If the mass of object C is $p\%$ of the mass of object B, what is the value of $\frac{p}{1{,}000}$?

Question 14:

For the positive quantities h, j, and k, **84% of** h is equivalent to **28% of** j, and j is equivalent to **40% of** k. What percentage of k is h?

Question 15:

The speeds of particles A, B, and C are a, b, and c meters per second, respectively. If the speed of particle A is **3900% greater** than the speed of particle C, and the speed of particle C is **99.98% less** than the speed of particle B, which expression best represents $a + b$ in terms of c?

A) $5{,}040\,c$
B) $5{,}039\,c$
C) $5{,}000\,c$
D) $50{,}400\,c$

Question 16:

How many liters of a 50% chlorine solution must be added to 16 liters of a 5% chlorine solution to obtain a 20% chlorine solution?

Question 17:

How many liters of a 40% chlorine solution must be added to 10 liters of a 10% chlorine solution to obtain a 25% chlorine solution?

PROBLEM-SOLVING & DATA ANALYSIS CONCEPT #1: PERCENTS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. C**
- ☐ **2. D**
- ☐ **3. C**
- ☐ **4. A**
- ☐ **5. D**
- ☐ **6. 500**
- ☐ **7. B**
- ☐ **8. 700**
- ☐ **9. 21.9**
- ☐ **10. 16**
- ☐ **11. B**
- ☐ **12. 5,200**
- ☐ **13. 437.5**
- ☐ **14. 13.3**
- ☐ **15. A**
- ☐ **16. 8**
- ☐ **17. 10**

Concept #2: Probability

💡**Tip:** *Make sure to look for the selected at random statement.* This tells you what you are choosing from and will be the number that goes in the DENOMINATOR.

<u>#</u> ⇐ What they want the probability of
 # ⇐ What you're selecting randomly from

Example:

The table below shows the distribution of objects in a collection by shape and color.

	Triangle	Rectangle
Blue	7	8
Red	5	6

If a triangle is selected at random, what is the probability that the selected triangle is red?

A. $\frac{5}{26}$

B. $\frac{5}{12}$

C. $\frac{5}{11}$

D. $\frac{5}{7}$

What we are selecting randomly from: _____

Number that goes in the denominator:_____

Conditional Probability

💡**Tip:** If the problem has a "given" statement, then you must take out at least one entire group from the total before calculating your probability.

Example:

A school club has 40 members, consisting of 24 juniors and 16 seniors. Among the juniors, 10 are on the debate team, while among the seniors, 6 are on the debate team.

If a club member is selected at random, what is the probability that the member is a junior, ***given they are not on the debate team***?

A. $\frac{14}{34}$

B. $\frac{14}{20}$

C. $\frac{14}{24}$

D. $\frac{14}{40}$

PRACTICE PROBLEMS:

Single Step "Part Out of Whole" Questions

Question 1:

Each face of a fair 10-sided die is labeled with a number from 1 through 10, with a different number on each face. If the die is rolled one time, what is the probability of rolling a 3?

A) $\frac{1}{10}$
B) $\frac{2}{10}$
C) $\frac{9}{10}$
D) $\frac{10}{10}$

Question 2:

A sleep study consisted of 72 participants, of which 57 participants each had an average of more than 150 minutes of rapid eye movement (REM) sleep per night. If a participant from this sleep study is selected at random, what is the probability of selecting a participant that had an average of more than 150 minutes of REM sleep per night?

A) $\frac{57}{100}$
B) $\frac{100}{72}$
C) $\frac{57}{72}$
D) $\frac{72}{57}$

Question 3:

A certain book has **300** pages, and **24** of these pages have an illustration. If one of the book's pages is selected at random, what is the probability of selecting a page with an illustration? *(Express your answer as a decimal or fraction, not as a percent.)*

Question 4:

84 students each voted for a new mascot. The results were as follows: 24 students voted for "Lion," and the other 60 students voted for other mascots. If one student is selected at random, what is the probability of selecting a student who voted for "Lion"?

A) $\frac{1}{7}$

B) $\frac{2}{7}$

C) $\frac{3}{7}$

D) $\frac{4}{7}$

Question 5:

For a particular machine that produces beads, 13 out of every 50 beads it produces have a defect. A bead produced by the machine will be selected at random. What is the probability of selecting a bead that has a defect?

A) $\frac{1}{650}$

B) $\frac{13}{50}$

C) $\frac{50}{13}$

D) $\frac{13}{100}$

Leftover Probability Questions

Question 6:

A movie theater has 400 customers, each located in theater A, B, or C. If one customer is selected at random, the probability that the selected customer is in theater A is 0.45 and the probability the selected customer is in theater B is 0.30. How many customers are located in theater C?

A) 25
B) 40
C) 100
D) 120

Question 7:

At a conference with 275 attendees, each attendee is assigned to group A, group B, or group C. If one attendee is selected at random, the probability of selecting an attendee who is in group A is 0.44, and the probability of selecting an attendee who is in group B is 0.24. How many attendees are in group C?

A) 33
B) 44
C) 88
D) 99

Conditional Probability Questions

Question 8:

A local organization has 130 members, categorized by age (less than 40 vs. at least 40) and whether they live east or west of a river. The table below shows the distribution:

	Live east of the river	Live west of the river	Total
Less than 40	12	14	26
At least 40	23	81	104
Total	35	95	130

If one member of this organization is selected at random, what is the probability that the selected member is at least 40 years old?

A) $\frac{26}{130}$

B) $\frac{35}{130}$

C) $\frac{95}{130}$

D) $\frac{104}{130}$

Question 9:

A science project tracked whether it rained on weekdays and weekend days for 10 weeks (70 days total). The results are summarized:

	Rain	No rain	Total
Weekdays	10	40	50
Weekend	5	15	20
Total	15	55	70

If one of the 55 days on which there was no rain is selected at random, what is the probability that the day was a weekend day?

A) $\frac{2}{11}$

B) $\frac{3}{11}$

C) $\frac{1}{7}$

D) $\frac{5}{11}$

Question 10:

A researcher surveyed 96 participants, classifying each participant by age (0–9 years, 10–19 years, or 20+ years) and by group (A, B, or C). The results are shown in the table below:

	0–9 years	10–19 years	20+ years	Total
Group A	3	17	11	31
Group B	2	9	20	31
Group C	1	3	30	34
Total	6	29	61	96

One of these 96 participants will be selected at random. What is the probability of selecting a participant who is in Group A, given that the participant is at least 10 years old?

A) $\frac{3}{24}$

B) $\frac{17}{35}$

C) $\frac{31}{96}$

D) $\frac{14}{45}$

Question 11:

A blood drive recorded the number of people with each blood type, further categorized by Rhesus factor ($+$ or $-$). Part of the results are shown below:

Rhesus Factor	A	B	AB	O
$+$	28	10	5	32
$-$	6	3	1	x

If **one Rhesus-negative ($-$) donor** is selected at random, and the probability that the donor has blood type **B** is $\frac{1}{4}$, what is the value of x?

Tricky Conditional Probability Questions

Question 12:

A city's population is distributed by age group as follows:

Age Group	Proportion
Less than 18 years	30%
18–40 years	25%
41–65 years	25%
Greater than 65	20%

If a person in this city is selected at random, which of the following is closest to the probability of selecting a person who is older than 65, given that the person is at least 18 years old?

A) 0.20

B) 0.26

C) 0.29

D) 0.40

Question 13:

A grove has 7 rows of birch trees and 5 rows of maple trees.

- Each row of birch trees has 10 trees that are 20 feet or taller and 4 trees that are shorter than 20 feet.
- Each row of maple trees has 12 trees that are 20 feet or taller and 3 trees that are shorter than 20 feet.

A tree from one of these rows will be selected at random. What is the probability of selecting a maple tree, **given** that the tree is 20 feet or taller?

A) $\frac{5}{13}$

B) $\frac{6}{13}$

C) $\frac{7}{13}$

D) $\frac{10}{13}$

Question 14:

An alumni association survey asked each high school graduate to select **one activity** for the association's next event. Some people responded by phone, and others responded by email. The table below shows the distribution of preferred activity, in percent, for each response type:

Activity	Phone	Email
Dance night	60%	50%
Football game	10%	20%
Potluck	20%	20%
Movie day	10%	10%
Total	100%	100%

The total number of **email** responses was **three times** the total number of **phone** responses.
If a person **who preferred Potluck** is selected at random, **what is the probability** that the person responded by **email**?

Question 15:

A new 3-day quiz show had the same **18** contestants answering **5** questions each day. Each contestant received 1 point for each correct answer. The number of contestants who earned each possible score (out of 5) on each day is shown in the table below:

Day	5/5	4/5	3/5	2/5	1/5	0/5	Total
Day 1	1	2	4	5	3	3	18
Day 2	2	3	4	4	3	2	18
Day 3	3	2	5	3	3	2	18
Total	6	7	13	12	9	7	54

No contestant received the same score on two different days. If a contestant is selected at random, what is the probability that the selected contestant received a score of **5** on **Day 2** or **Day 3**, **given** that the selected contestant received a score of **5** on exactly **one** of the three days?

PROBLEM-SOLVING & DATA ANALYSIS CONCEPT #2: PROBABILITY ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. A**
- ☐ **2. C**
- ☐ **3. 2/25 or 0.08**
- ☐ **4. B**
- ☐ **5. B**
- ☐ **6. C**
- ☐ **7. C**
- ☐ **8. D**
- ☐ **9. B**
- ☐ **10. D**
- ☐ **11. 2**
- ☐ **12. C**
- ☐ **13. B**
- ☐ **14. 0.75**
- ☐ **15. 5/6**

Concept #3: Ratios

WHAT YOU NEED TO KNOW

Ratio: **3 parts water to 2 parts oil**

How can you write that ratio?

1. _____ or **2.**_____

You will often use the fraction format in order to solve by setting up a proportion.

💡 **Tip:** On the SAT, it is important to know how many parts TOTAL you have in a ratio.

Example: *There are 3 parts water to 2 parts oil. If a total of 11,386 gallons of water and oil are used, how many gallons of oil are needed?*

How to set up the proportion:

_____ = _____

Rates are also ratios. If you see "per" in the problem, you can probably write the relationship as a ratio!

Example: The cheetah runs 55 miles per hour.

Ratio: _____

💡**Tip:** A popular ratio on this test is the Density Formula! Please write it below and make sure you know it!

Density Formula

PRACTICE PROBLEMS

Simple Ratios/Proportions

Question 1:

A certain pigeon species can fly at an average speed of 18 meters per second when in continuous flight. At this rate, how many meters would this pigeon species fly in 5 seconds?

A) 90

B) 72

C) 36

D) 45

Question 2:

A customer spent $45 to purchase oranges at $5 per pound. How many pounds of oranges did the customer purchase?

Question 3:

An object travels at a constant speed of **16 centimeters per second**. At this speed, what is the time, in seconds, that it would take for the object to travel **144 centimeters**?

A) 9

B) 96

C) 160

D) 230

Question 4:

The ratio of green tiles to blue tiles in a piece of artwork is **3 to 1**. If there are **12** blue tiles in the piece of artwork, how many green tiles are there?

Ratio Adjustment Questions

Question 5:

For a certain rectangular region, the ratio of its length to its width is **9 to 4**. If the width of the rectangular region increases by **4** units, how must the length change to maintain this ratio?

A) It must decrease by 5 units.

B) It must increase by 9 units.

C) It must increase by 4 units.

D) It must decrease by 1 unit.

Question 6:

For a certain rectangular region, the ratio of its length to its width is **12 to 5**. If the width of the rectangular region increases by **5** units, how must the length change to maintain this ratio?

A) It must decrease by 5 units.
B) It must increase by 5 units.
C) It must increase by 12 units.
D) It must decrease by 12 units.

Density Formula Questions

Question 7:

A sample of oak has a density of **850** kilograms per cubic meter. The sample is in the shape of a cube, where each edge has a length of **0.75** meters. To the nearest whole number, what is the mass, in kilograms, of this sample?

A) 215
B) 359
C) 687
D) 892

Question 8:

A sample of pine wood has a density of **600** kilograms per cubic meter. The sample is in the shape of a cube, and its mass is **432** kilograms. To the nearest hundredth, what is the volume, in cubic meters, of this sample?

A) 0.56
B) 0.72
C) 0.89
D) 1.12

Advanced Ratios/Proportions

Question 9:

A proposal for a new community center was included on an election ballot. A radio show stated that **4 times** as many people voted in favor of the proposal as people who voted against it. A social media post reported that **24,000** more people voted in favor of the proposal than voted against it. Based on these data, how many people voted against the proposal?

A) 6,000
B) 8,000
C) 12,000
D) 24,000

Question 10:

The ratio **6 to** x is equivalent to the ratio y **to 9**. Which equation represents x in terms of y?

A) $x = \frac{6y}{9}$
B) $x = \frac{9y}{6}$
C) $x = \frac{y}{54}$
D) $x = \frac{54}{y}$

Question 11:

Candidate	Votes
Vanessa Cruz	528
Nick Roberts	372

The table shows the results of a poll. A total of **900** voters selected at random were asked which candidate they would vote for in the upcoming election. According to the poll, if **7,200** people vote in the election, by how many votes would Vanessa Cruz be expected to win?

A) 176
B) 956
C) 1,248
D) 2,816

PROBLEM-SOLVING & DATA ANALYSIS CONCEPT #3: RATIOS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. A**
- ☐ **2. 9**
- ☐ **3. A**
- ☐ **4. 36**
- ☐ **5. B**
- ☐ **6. C**
- ☐ **7. B**
- ☐ **8. B**
- ☐ **9. B**
- ☐ **10. D**
- ☐ **11. C**

Concept #4: Conversion Problems

Converting units is basically multiplying one ratio by another.

Example: If I want to convert miles to kilometers, then I want miles to cancel.

Q: How do I convert 20 miles into kilometers? (1 km = 0.62 mi)

$$\frac{20\ miles}{} * \ \_\_\_\_$$

Circle One: **Top / Bottom**

💡**Tip:** You may need to convert a conversion factor to the proper dimension first, by either squaring or cubing it if your problem is in square or cubed units.

Example:

A certain town has an area of 4.36 square miles. What is the area, in square yards, of this town? $(1\ \textbf{mile} = 1{,}760\ \textbf{yards})$

A. 404

B. $7{,}674$

C. $710{,}459$

D. $13{,}505{,}536$

PRACTICE PROBLEMS

Simple Unit Conversion

Question 1:

How many centimeters are equivalent to **82** meters?

$(1 \text{ meter } = 100 \text{ centimeters})$

Question 2:

A printer produces posters at a constant rate of **36** posters per minute. At what rate, in posters per hour, does the printer produce the posters?

Question 3:

An object moves at a speed of $\frac{3}{20}$ feet per second. $(3 \text{ feet} = 1 \text{ yard})$

What is this speed, in yards per second?

A) 20

B) $\frac{3}{20}$

C) $\frac{9}{20}$

D) $\frac{1}{20}$

Squared Unit Conversions

Question 4:

A certain town has an area of **2.75** square miles. What is the area, in **square yards**, of this town?

$(1 \text{ mile} = 1{,}760 \text{ yards})$

A) 501

B) 4,840

C) 6,178

D) 8,518,400

Question 5:

A certain park has an area of **7,744,000** square yards. What is the area, in **square miles**, of this park?

$(1 \text{ mile} = 1{,}760 \text{ yards})$

A) 1.25

B) 2.50

C) 4,400

D) 25

Question 6:

An area of **62.00** square nautical miles is equivalent to k square kilometers. To the nearest tenth, what is the value of k?

$(1 \text{ nautical mile} = 1.852 \text{ kilometers})$

Question 7:

The area of a rectangular region is increasing at a rate of **350** square feet per hour. Which of the following is closest to this rate in **square meters per minute**?

$(\text{Use } 1 \text{ meter } = 3.28 \text{ feet.})$

A) 0.54

B) 1.28

C) 15.31

D) 26.03

Tricky Conversion Questions

Question 8:

On average, a certain tree grows **92 centimeters every n months**. At this rate, which expression represents the number of centimeters, on average, the tree grows every r **years**?

$$(1 \text{ year} = 12 \text{ months})$$

A) $\frac{92r}{12n}$

B) $\frac{1104r}{n}$

C) $\frac{12n}{92r}$

D) $\frac{n}{1104r}$

Question 9:

A container has a volume of **1.20** cubic meters. Which of the following is **closest** to this volume, in **quarts**?

(Note: 100.00 centimeters = 1 meter. Use 946.35 cubic centimeters = 1.00 quart.)

A) 0.12

B) 12.67

C) 1,268

D) 10,592

Question 10:

There are 640 acres in 1 square mile. The area of a forest is increasing at a rate of 1 acre per decade. Which of the following is closest to the rate at which the area of the forest is increasing, in **square kilometers** per decade? (Use 1 kilometer = 0.62 mile.)

A) 0.0006

B) 0.0010

C) 0.0025

D) 0.0041

Question 11:

A rocket's acceleration is measured at a rate of 3.45 meters per second squared. What is this rate in kilometers per hour squared? (Use 1 kilometer = 1000 meters.)

PROBLEM-SOLVING & DATA ANALYSIS CONCEPT #4: CONVERSION PROBLEMS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. 8,200**
- ☐ **2. 2,160**
- ☐ **3. D**
- ☐ **4. D**
- ☐ **5. B**
- ☐ **6. 212.7**
- ☐ **7. A**
- ☐ **8. B**
- ☐ **9. C**
- ☐ **10. D**
- ☐ **11. 44,712**

✨How can you make the most out of this workbook? ✨

👉For walkthrough videos, strategies, quizzes, and extra resources that support each lesson in this workbook, please sign up for our Digital SAT Math Course!

👉For individualized support, please check out our Information Pack to learn more about our private tutoring options and Request a Consultation!

👉Did you know we also have a Digital SAT English Course? If you're trying to improve your score in both areas of the test, check it out!

Concept #5: Statistics

WHAT YOU NEED TO KNOW

Mean:

Median:

Mode:

Range:

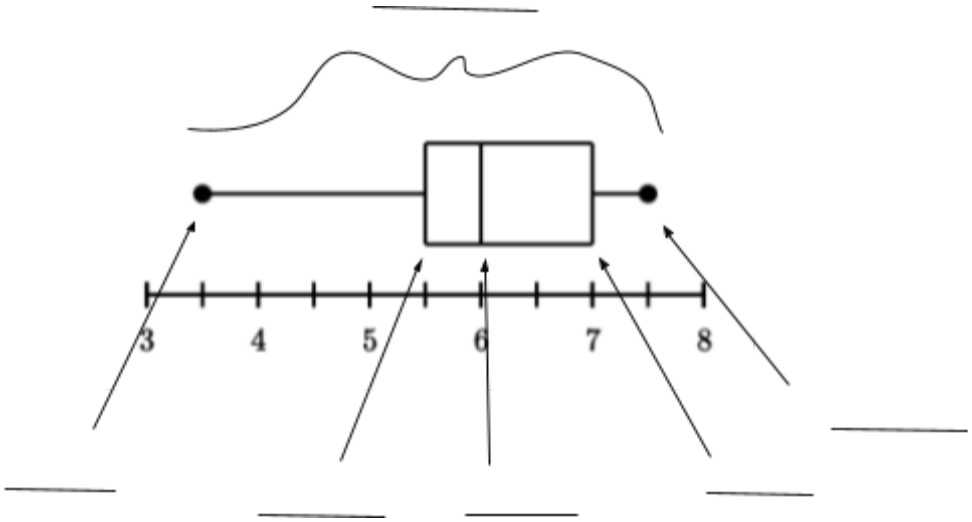
Outliers affect the (circle one): **Mean / Median**

To calculate the median, first put the data in (circle one): **Ascending / Descending** order.

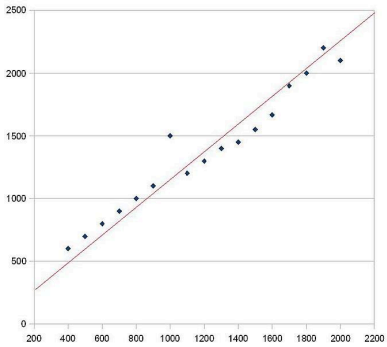
The greater the standard deviation, the more (circle one) **Spread Out / Concentrated** the data is.

Data that has a higher Standard Deviation:	**Data that has a lower Standard Deviation:**
20, 30, 40, 50, 60, 70	**40, 50, 50, 50, 50, 60**

Parts of a box plot:



Scatterplots



In a scatter plot, the **line of best fit** represents the (circle one): **ACTUAL / ESTIMATES**

In a scatter plot, the **dots** represent the (circle one): **ACTUAL / ESTIMATES**

If dot plots/bar graphs are **symmetrical** about the same value, they have the same (circle all that apply): MEAN, MEDIAN, STANDARD DEVIATION

Evaluating Statistical Claims

Whatever the study takes a random sample from is what a study can be generalized to.

Example:
A study takes a random sample of 250 7th graders from Broome county in the state of New York to survey them on something. What can this study be generalized to?
A) All 7th grade students
B) All students in the state of New York
C) All 7th grade students in the state of New York
D) All 7th grade students in Broome Country

PRACTICE PROBLEMS

Central Tendency Questions

Question 1:

The frequency table summarizes the 62 data values in a data set. What is the maximum data value in the data set?

Data value	Frequency
5	4
6	5
7	7
8	10
9	12
10	11
11	8
12	0
13	5

Question 2:

15, 17, 22, 26, 30

What is the median of the data set shown?

Question 3:

The table shows the frequency of values in a data set.

Value	Frequency
11	7
18	2
25	5
32	9

What is the minimum value of the data set?

A) 2

B) 7

C) 11

D) 18

Question 4:

The list gives the number of individuals in each of **6** groups of ring-tailed lemurs.

12, 5, 19, 14, 8, 7

What is the **range** of the numbers of individuals for the **6** groups of ring-tailed lemurs?

Question 5:

4, 7, 10, 14, 25

What is the **mean** of the data shown?

A) 10

B) 12

C) 14

D) 15

Question 6:

The table shows the amount of time it took a participant in a study to complete each of 5 tasks. What was the mean time, in minutes, for this participant to complete a task?

Task	Time (minutes)
A	5
B	10
C	15
D	10
E	10

A) 8.5

B) 9.2

C) 10.0

D) 11.3

Question 7:

An environmental scientist is investigating the volatility of **200** organic compounds by finding the boiling point of each compound. The mean boiling point of all **200** compounds is **185** degrees Celsius. The scientist classifies each of these compounds as either semivolatile or volatile. Of the **200** compounds, **60** compounds were classified as semivolatile, and these **60** compounds have a mean boiling point of **340** degrees Celsius. The remaining **140** compounds were classified as volatile. What is the mean boiling point, in degrees Celsius, of the **140** compounds classified as volatile compounds?

Comparing Data Sets

Question 8:

Data set A: 3, 3, 4, 6, 7, 9, 10, 12, 13, 15, 15

Data set B: 4, 5, 6, 8, 8, 9, 10, 11, 12, 14, 14

Data set A and data set B each have **11** values. Which of the following statements best compares the median of data set A and the median of data set B?

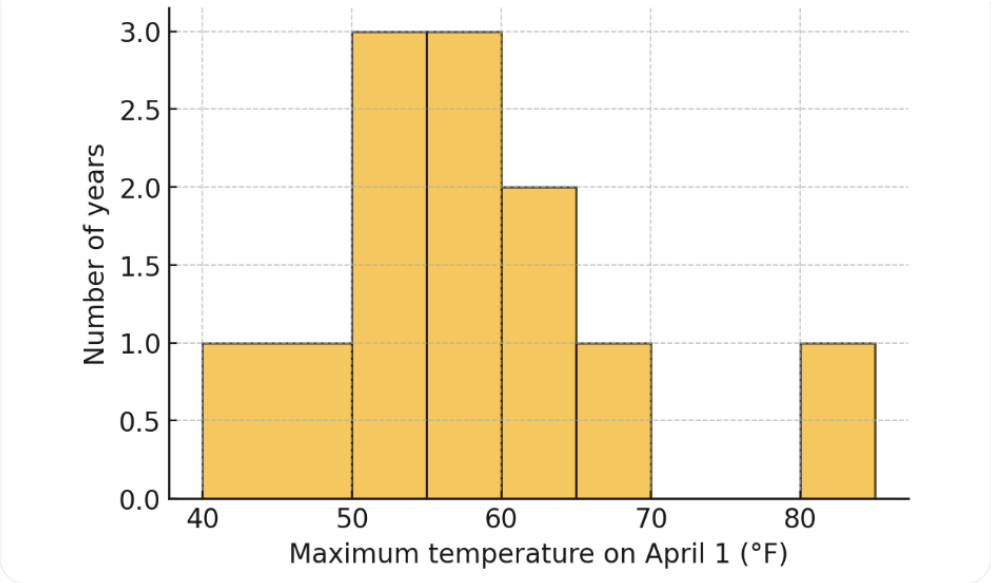
A) The median of data set A is greater than the median of data set B.

B) The median of data set A is less than the median of data set B.

C) The medians of data sets A and B are equal.

D) There is not enough information to compare the medians.

Question 9:



The maximum temperature on April 1, in degrees Fahrenheit (°F), was recorded each year at a certain weather station for **11** years. The histogram summarizes the recorded data set.

The temperature of **80°F** is removed from this data set to create a new data set of **10** temperatures. Which of the following statements must be true?

I. The mean of the new data set is less than the mean of the original data set.

II. The median of the new data set is less than the median of the original data set.

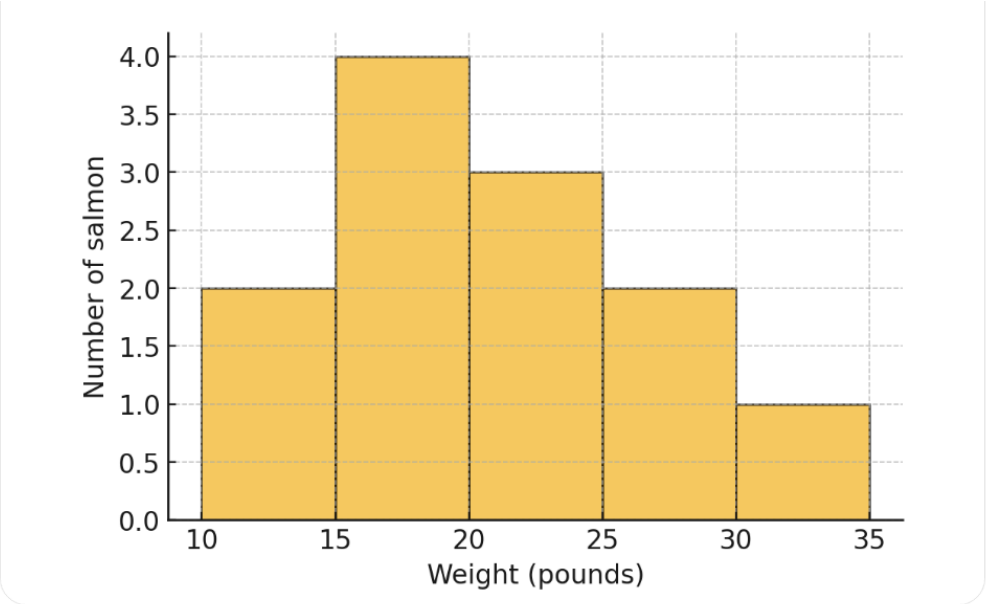
A) I only

B) II only

C) I and II

D) Neither I nor II

Question 10:



The histogram summarizes a data set of the weights, in pounds, of **12** salmon. If an additional weight of **78** pounds is added to the original data set to create a new data set of **13** weights of salmon, which of the following measures must be greater for the new data set than for the original data set?

I. The median

II. The mean

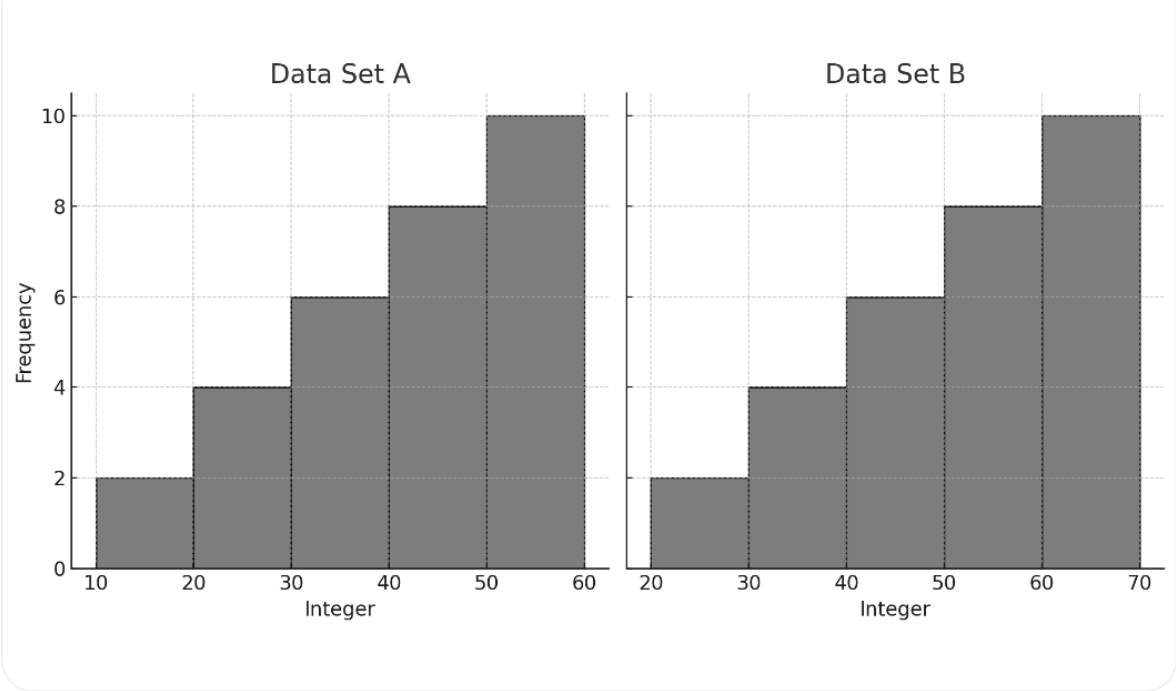
A) I only

B) II only

C) I and II

D) Neither I nor II

Question 11:



For each of the histograms, the first interval represents the frequency of integers greater than or equal to 10, but less than 20. The second interval represents the frequency of integers greater than or equal to 20, but less than 30, and so on.

What is the smallest possible difference between the mean of data set A and the mean of data set B?

A) 0

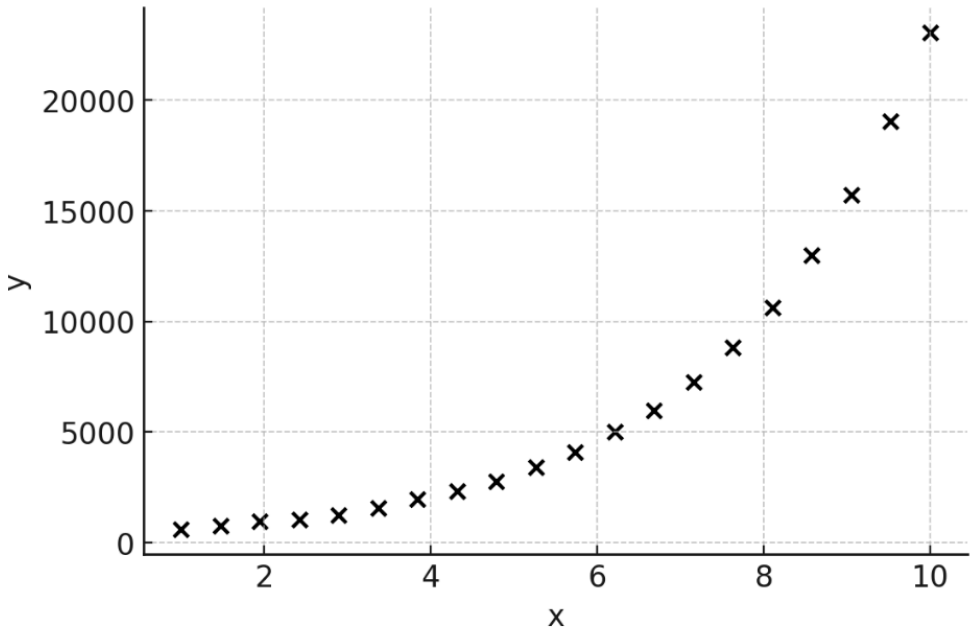
B) 1

C) 10

D) 23

Line of Best Fit Questions

Question 12:



The scatterplot shows the relationship between two variables, **x** and **y**.

(See the scatterplot above.)

Which of the following equations is the most appropriate model for the data shown?

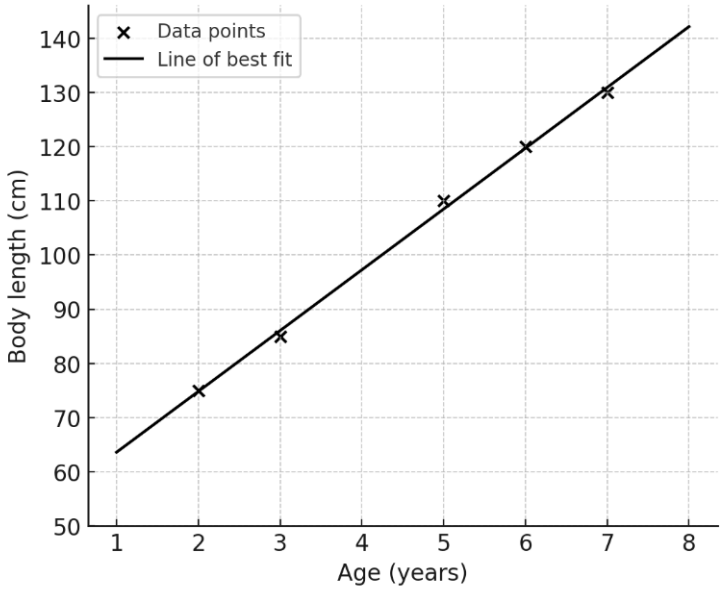
A) $y = -400 + 1.4x$

B) $y = 400 - 1.4x$

C) $y = -400(1.5)^x$

D) $y = 400(1.5)^x$

Question 13:



The scatterplot shows **5** measurements of the body length, in **centimeters (cm)**, of a New Zealand fur seal from an age of **2 years to 7 years old**. A line of best fit is also shown.

For a New Zealand fur seal at an age of **4 years old**, what is the body length predicted by the line of best fit, to the nearest **10 cm**?

Standard Deviation Questions

Question 14:

Data set A: **6, 9, 12, 18, 22, 28**

Data set B: **7, 11, 14, 19, 23, k**

Data sets A and B each consist of **6** values as shown, where **k** is a constant. If the standard deviation of data set A is greater than the standard deviation of data set B, which of the following could be the value of **k**?

I. 26

II. 30

III. 35

A) I only

B) II only

C) III only

D) II or III

Question 15:

Each of the following frequency tables represents a data set. Which of these frequency tables represents the data set with the **smallest standard deviation**?

Table A

Value	Frequency
3	0
4	8
5	14
6	20
7	14
8	0

Table B

Value	Frequency
3	10
4	12
5	16
6	12
7	10

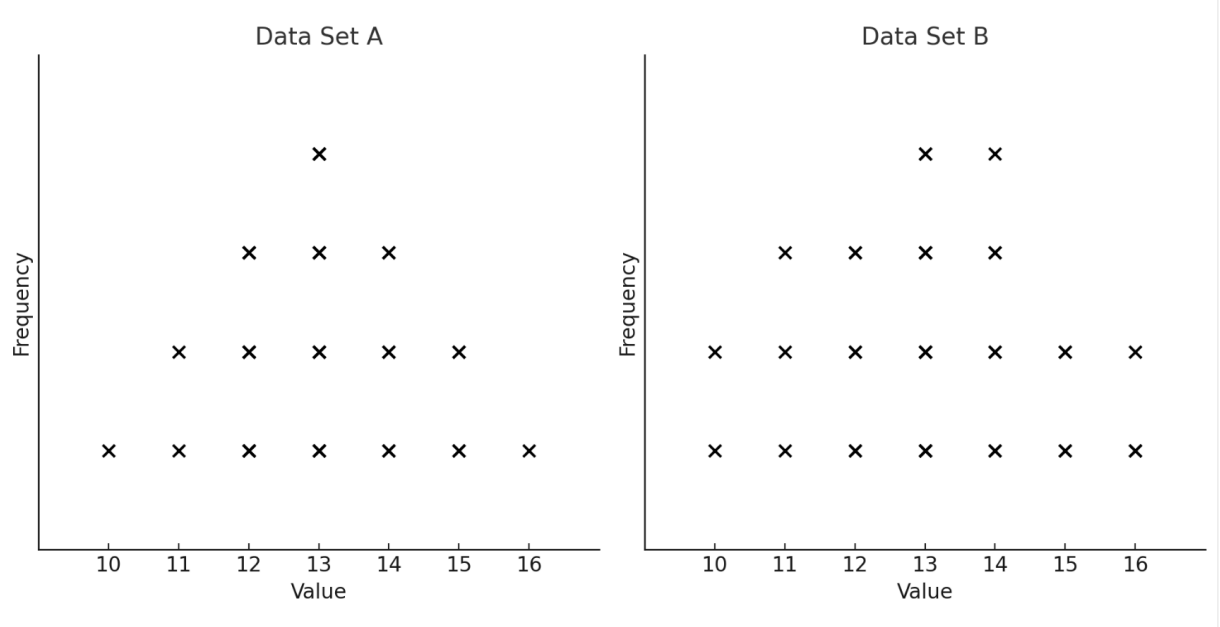
Table C

Value	Frequency
3	10
4	10
5	10
6	10
7	10

Table D

Value	Frequency
3	14
4	11
5	8
6	11
7	14

Question 16:



Which of the following statements must be true?

I. The median of data set A is equal to the median of data set B.

II. The standard deviation of data set A is equal to the standard deviation of data set B.

A) I only

B) II only

C) I and II

D) Neither I nor II

Question 17:

The values in data sets X and Y are shown in the table below.

Data set X	12	12	13	13	14	15	15	16	16
Data set Y	3	3	4	4	5	6	6	7	7

The standard deviation of data set X is **q**, and the standard deviation of data set Y is **s**. Which of the following statements about the standard deviation of the data sets is true?

A) $q < s$

B) $q > s$

C) $q = s$

D) The relationship between q and s cannot be determined.

Evaluating Statistical Claims

Question 18:

A sample of 40 fourth-grade students was selected at random from a certain school. The 40 students completed a survey about the morning announcements, and 32 thought the announcements were helpful. Which of the following is the largest population to which the results of the survey can be applied?

A. The 40 students who were surveyed

B. All fourth-grade students at the school

C. All students at the school

D. All fourth-grade students in the county in which the school is located

Question 19:

A trivia tournament organizer wanted to study the relationship between the number of points a team scores in a trivia round and the number of hours that a team practices each week. For the study, the organizer selected 55 teams at random from all trivia teams in a certain tournament. The table displays the information for the 40 teams in the sample that practiced for at least 3 hours per week.

Hours practiced	Number of points per round		
	6 to 13 points	**14 or more points**	**Total**
3 to 5 hours	6	4	10
More than 5 hours	4	26	30
Total	**10**	**30**	**40**

Which of the following is the largest population to which the results of the study can be generalized?

A. All trivia teams in the tournament that scored 14 or more points in the round

B. The 55 trivia teams in the sample

C. The 40 trivia teams in the sample that practiced for at least 3 hours per week

D. All trivia teams in the tournament

PROBLEM-SOLVING & DATA ANALYSIS CONCEPT #5: STATISTICS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. 13**
- ☐ **2. 22**
- ☐ **3. 11**
- ☐ **4. 14**
- ☐ **5. B**
- ☐ **6. C**
- ☐ **7. 118.57**
- ☐ **8. C**
- ☐ **9. A**
- ☐ **10. B**
- ☐ **11. B**
- ☐ **12. D**
- ☐ **13. 100**
- ☐ **14. A**
- ☐ **15. A**
- ☐ **16. A**
- ☐ **17. C**
- ☐ **18. B**
- ☐ **19. D**

Concept #6: Margin of Error

WHAT YOU NEED TO KNOW

How to decrease Margin of Error?

A) Change the tool you're using
B) Try more experiments
C) Increase the sample size
D) Decrease the sample size

Answer: _____

How to calculate the Margin of Error?

If there is a 3% margin of error and it was estimated that 21% of a 100,000 population has susceptibility to a disease, then you will calculate:

21% - 3% = **18%** of the population for the **lower limit**
and
21% + 3% = **24%** of the population for the **upper limit**

100,000(.18) = _____ and 100,000(.24) = _____

so, the population that is susceptible to the disease is likely to be anywhere between these two numbers given the margin of error.
Please note it is possible, but unlikely, for the actual population to be outside this range

PRACTICE PROBLEMS

Sample Sizes

Question 1:

A research manager selected **2 random samples** of ovens of a certain type to estimate the average amount of time this type of oven takes to preheat to **350 degrees Fahrenheit (°F)**. The research manager recorded the amount of time, in minutes, each oven takes to preheat to **350°F**. Based on the first sample, the research manager estimated that this type of oven takes an average of **14.8 minutes** to preheat to **350°F**, with an associated margin of error of **0.8 minutes**. Based on the second sample, the research manager estimated that this type of oven takes an average of **15.5 minutes** to preheat to **350°F**, with an associated margin of error of **2.5 minutes**.

Assuming the margins of error were calculated the same way, which of the following best explains why the first sample obtained a smaller margin of error than the second sample?

A) The first sample contained more ovens than the second sample.
B) The first sample contained fewer ovens than the second sample.
C) The first sample took more time on average to preheat to **350°F** than the second sample.
D) The first sample took less time on average to preheat to **350°F** than the second sample.

Question 2:

The manager of a gym selected a sample of **150** members at random to estimate the percentage of the gym's members that would continue to pay for a membership if the price increased. From the survey, the manager estimates that **78%** of the gym's members would continue to pay for a membership if the price increased, with an associated margin of error of **5.8%**. If the survey is repeated with a random sample of **300** members and the results are calculated in the same way, which of the following will be the most likely effect of using the larger random sample compared to the smaller random sample?

A) The margin of error will be lower.
B) The margin of error will be higher.
C) The estimate of the percentage of the gym's members that would continue to pay for a membership if the price increased will be lower.
D) The estimate of the percentage of the gym's members that would continue to pay for a membership if the price increased will be higher.

Using Margin of Error in Calculations for Conclusions

Question 3:

A group of **50** employees selected at random from all the employees at a company were surveyed about the number of books they read last year. From the data collected, it was estimated that the mean number of books employees at the company read last year is **7.4**, with an associated margin of error of **1.3**. Which of the following is the most appropriate conclusion?

A) It is plausible that the actual mean number of books employees at the company read last year is less than **6.1**.
B) It is plausible that the actual mean number of books employees at the company read last year is between **6.1** and **8.7**.
C) It is plausible that every employee at the company read between **7.4** and **10.0** books last year.
D) It is plausible that the actual mean number of books employees at the company read last year is greater than **8.7**.

Question 4:

A machine fills bags with approximately **15** ounces of sugar. To test the accuracy of the filling process, **150** bags of sugar were selected at random and weighed. Based on the sample, it is estimated that the average weight of all bags of sugar filled by the machine in an **8-hour** period is **14.72** ounces, with an associated margin of error of **0.3** ounces. Which of the following is the best interpretation of this estimate?

A) Plausible values for the average weight of all bags of sugar filled by the machine are between **14.42** ounces and **15.02** ounces.
B) Plausible values for the average weight of all bags of sugar filled by the machine are less than **14.42** ounces or greater than **15.02** ounces.
C) The average weight of all bags of sugar filled by the machine is greater than **14.85** ounces.
D) The average weight of all bags of sugar filled by the machine is less than **14.85** ounces.

Question 5:

A sample consisting of **850** adults who own televisions was selected at random for a study. Based on the sample, it is estimated that **36%** of all adults who own televisions use their televisions to watch news programs, with an associated margin of error of **4.2%**. Which of the following is the most plausible conclusion about all adults who own televisions?

A. More than **40.2%** of all adults who own televisions use their televisions to watch news programs.

B. Between **31.8%** and **40.2%** of all adults who own televisions use their televisions to watch news programs.

C. Since the sample included adults who own televisions and not just those who use their televisions to watch news programs, no conclusion can be made.

D. Since the sample did not include all the people who watch news programs, no conclusion can be made.

Making Conclusions

Question 6:

A park ranger asked a random sample of visitors how far they hiked during their visit. Based on the responses, the estimated mean was found to be **5.2** miles, with an associated margin of error of **0.6** miles. Which of the following is the best conclusion from these data?

A. It is likely that all visitors hiked between **4.6** and **5.8** miles.

B. It is likely that most visitors hiked exactly **5.2** miles.

C. It is not possible that any visitor hiked less than **3.5** miles.

D. It is plausible that the mean distance hiked for all visitors is between **4.6** and **5.8** miles.

Question 7:

In a study of cell phone use, **850** randomly selected US teens were asked how often they talked on a cell phone and about their texting behavior. The data are summarized in the table below. Based on the data from the study, an estimate of the percent of US teens who are heavy texters is **28%**, and the associated margin of error is **2%**. Which of the following is a correct statement based on the given margin of error?

Texting behavior	Talks on cell phone daily	Does not talk on cell phone daily	Total
Light	120	155	275
Medium	145	170	315
Heavy	160	100	260
Total	**425**	**425**	**850**

A. Approximately **2%** of the teens in the study who are classified as heavy texters are not really heavy texters.

B. It is not possible that the percent of all US teens who are heavy texters is less than **26%**.

C. The percent of all US teens who are heavy texters is **30%**.

D. It is doubtful that the percent of all US teens who are heavy texters is **33%**.

Question 8:

A study was done on the weights of different types of fish in a lake. A random sample of fish were caught and marked in order to ensure that none were weighed more than once. The sample contained **200** largemouth bass, of which **25%** weighed more than **3 pounds**. Which of the following conclusions is best supported by the sample data?

A. The majority of all fish in the lake weigh less than **3 pounds**.

B. The average weight of all fish in the lake is approximately **3 pounds**.

C. Approximately **25%** of all fish in the lake weigh more than **3 pounds**.

D. Approximately **25%** of all largemouth bass in the lake weigh more than **3 pounds**.

PROBLEM-SOLVING & DATA ANALYSIS CONCEPT #6: MARGIN OF ERROR ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. A**
- ☐ **2. A**
- ☐ **3. B**
- ☐ **4. A**
- ☐ **5. B**
- ☐ **6. D**
- ☐ **7. D**
- ☐ **8. D**

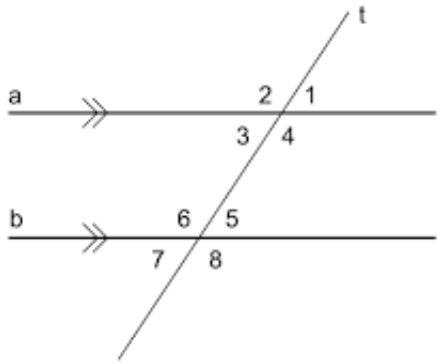
Chapter 6
Geometry and Trigonometry

Geometry & Trigonometry

Concept #1: Angle Chasing

WHAT YOU NEED TO KNOW

Parallel Lines



Corresponding Angles: ∠____ and ∠____ ; ∠____ and ∠____

Alternate Interior: ∠____ and ∠____ ; ∠____ and ∠____

Alternate Exterior: ∠____ and ∠____ ; ∠____ and ∠____

Degrees in a line: _____

Degrees in a triangle: _____

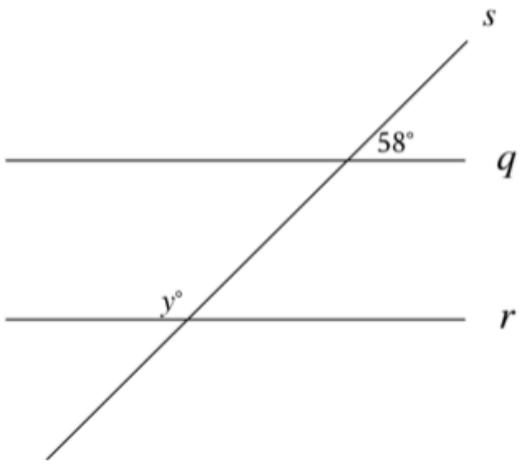
Degrees in a quadrilateral: _____

Formula for finding degrees in a n-sided polygon: _____

👉Example: How many degrees are in a 76-sided polygon?

PRACTICE PROBLEMS

Question 1:



Note: Figure not drawn to scale.

In the figure shown, line q is parallel to line r, and both lines are intersected by line s. If $y = 2x + 16$, what is the value of x?

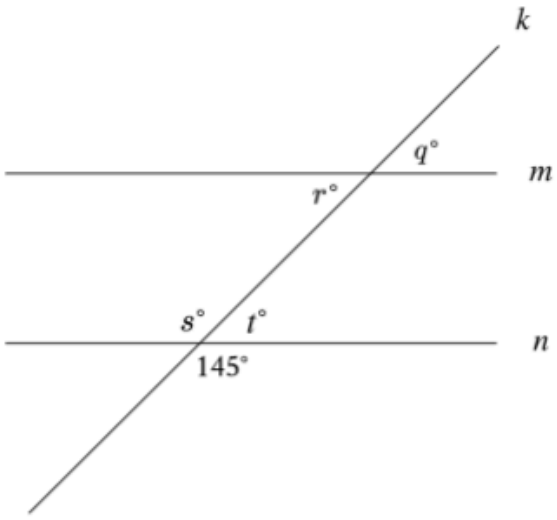
A) 28
B) 37
C) 45
D) 53

Question 2:

In $\triangle PQR$, the measures of both $\angle P$ and $\angle Q$ are equal, and the measure of $\angle R$ is $96°$. What is the measure of $\angle P$?

A) $42°$
B) $48°$
C) $84°$
D) $96°$

Question 3:

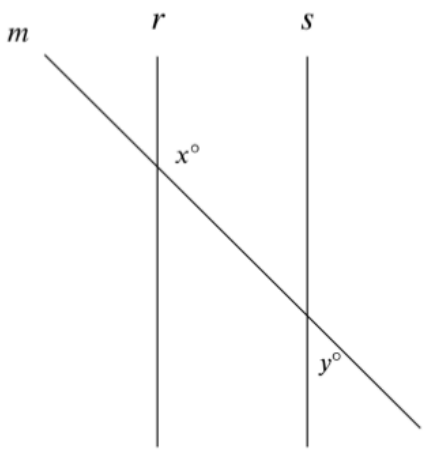


Note: Figure not drawn to scale.

In the figure shown, line m is parallel to line n, and line k intersects both lines. Which of the following statements is true?

A) $q = 145$
B) $r = 145$
C) $s = 145$
D) $t = 145$

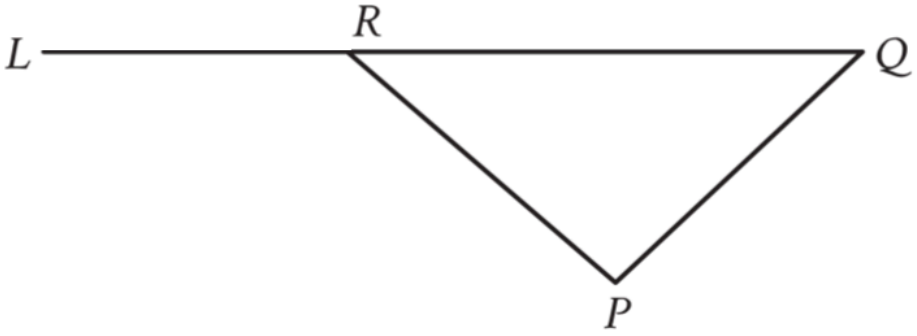
Question 4:



Note: Figure not drawn to scale.

In the figure shown, line r is parallel to line s, and both lines are intersected by line m. If $x = 3k + 18$, $y = 6k + 30$, and $9k = a$, where a is a constant, what is the value of a?

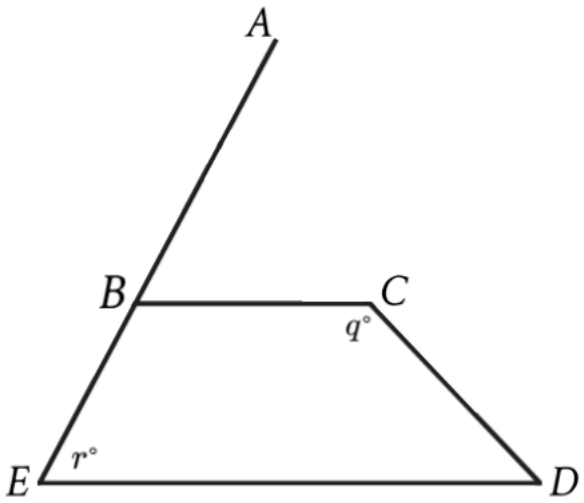
Question 5:



Note: Figure not drawn to scale.

$\triangle PQR$, QR is extended to point L. The measure of $\angle PQR$ is $42°$, and the measure of $\angle PRL$ is $116°$. What is the measure of $\angle QPR$ in degrees?

Question 6:



Note: Figure not drawn to scale.

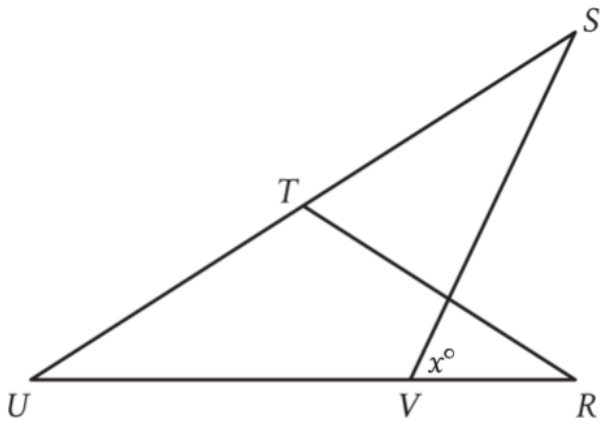
In the figure shown, point B lies on segment AE, and segment BC is parallel to segment DE. The measure of angle D is $32°$, and the measure of angle ABC is $68°$. What is the value of $q - r$?

Question 7:

Triangles ABC and XYZ are graphed in the xy-plane. Triangle ABC has vertices A, B, and C at $(1, 4)$, $(1, 8)$, and $(5, 4)$, respectively. Triangle XYZ has vertices X, Y, and Z at $(1, 4)$, $(1, 8 + n)$, and $(5 + n, 4)$, respectively, where n is a positive constant. If the measure of $\angle B$ is $w°$, what is the measure of $\angle Z$?

A) $(90 - w)°$
B) $(90 - w + n)°$
C) $(90 + w - n)°$
D) $(90 + w + n)°$

Question 8:



Note: Figure not drawn to scale.

In the figure, $RT = TU$. The measure of $\angle VST$ is $35°$, and the measure of $\angle RTU$ is $98°$. What is the value of x?

Question 9:

In $\triangle LMN$, the measure of angle L is $(2a - 7)°$, the measure of angle M is $(5a - 23)°$, and the measure of angle N is $(3b)°$. If side MN is extended through N to point K, and the measure of angle LNK is $(a + b)°$, what is the value of $a + b$?

Question 10:

Two lines intersect at exactly one point, forming two acute angles and two obtuse angles. The measure of one of these angles is $(7x - 70)°$. Which of the following could **NOT** be the sum of the measures of any two of these angles?

A) $180°$
B) $(14x - 140)°$
C) $(-14x + 140)°$
D) $(-14x + 500)°$

GEOMETRY & TRIGONOMETRY CONCEPT #1: ANGLE CHASING ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. D**
- ☐ **2. A**
- ☐ **3. C**
- ☐ **4. 132**
- ☐ **5. 74**
- ☐ **6. 80**
- ☐ **7. A** (it's also the only answer option without an "n"!)
- ☐ **8. 76**
- ☐ **9. 54** (note: once you get two equations for two unknowns, Desmos can solve the system if you use x and y instead!)
- ☐ **10. C**

Concept #2: Area and Perimeter

WHAT YOU NEED TO KNOW

To calculate the perimeter, you _____ the sides.

Circumference of a circle (Perimeter): _____

Area of a Triangle: _____

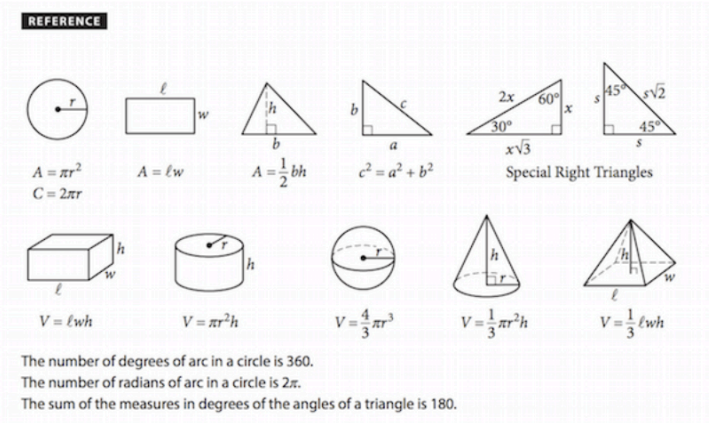
Area of a Square: _____

Area of a Rectangle: _____

Area of a Circle: _____

Area of a Parallelogram: _____

All of this can be found on the reference table, which is located in the upper right corner of the Bluebook screen:



PRACTICE PROBLEMS

Question 1:

A rectangle has a length that is 12 times its width. The function $A(x) = (12x)(x)$ represents this situation, where $A(x)$ is the area, in square feet, of the rectangle and $A(x) > 0$. Which of the following is the best interpretation of $12x$ in this context?

A) The width of the rectangle, in feet

B) The length of the rectangle, in feet

C) The area of the rectangle, in square feet

D) The difference between the length and the width of the rectangle, in feet

Question 2:

Triangle T has an area of 60 square centimeters (cm^2). Square S has side lengths of 6 cm. What is the total area of triangle T and square S, in cm^2?

A) 36

B) 72

C) 96

D) 108

Question 3:

Erin put up wooden fencing along each edge of a rectangular farm with a length of m feet and a width of n feet. Erin put up a total length of 580 feet of wooden fencing. Which equation represents this situation?

A) $mn = 580$
B) $2m = 580$
C) $m + n = 580$
D) $2m + 2n = 580$

Question 4:

The area of a triangle is 120 square centimeters. The length of the base of the triangle is 8 centimeters greater than the height of the triangle. What is the height, in centimeters, of the triangle?

A) 10
B) 12
C) 15
D) 20

Question 5:

The length of one side of a rectangle is 34 units longer than the length of another side. The rectangle's area is 800 square units. What is the length, in units, of the longer of these two sides?

Question 6:

A rectangular circuit board is divided into 270 square regions. Each of these regions has a length of L units. The length of the circuit board is 1.2 times its width. If the width of the circuit board is Lx units, what is the value of x?

Question 7:

In the xy-plane, the line segment with endpoints $(-4, 5)$ and $(10, -7)$ represents one of the legs of a right triangle. The area of this triangle is $54\sqrt{85}$ square units. What is the length, in units, of the other leg of this triangle?

A) 54
B) $54\sqrt{85}$
C) $27\sqrt{85}$
D) 27

GEOMETRY & TRIGONOMETRY CONCEPT #2: AREA AND PERIMETER ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. B**
- ☐ **2. C**
- ☐ **3. D**
- ☐ **4. B**
- ☐ **5. 50**
- ☐ **6. 15**
- ☐ **7. A**

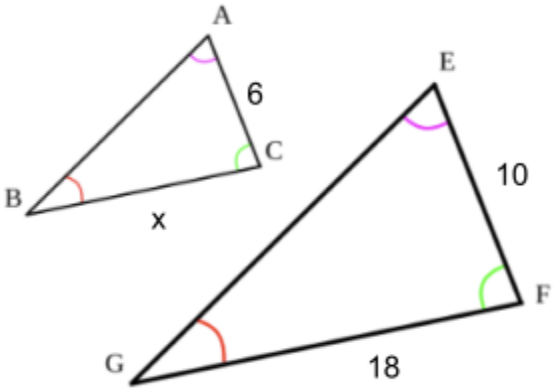
Concept #3: Similar Figures

WHAT YOU NEED TO KNOW

Similar figures have the **same angle measures**, but are different **sizes in proportion** to each other. Many problems involve triangles, but the same rules will apply to any shape.

Example
Triangles ABC and EGF are similar, where A corresponds to E, G corresponds to B and C corresponds to F. AC = 6, EF = 10, and GF = 18.

Find the length of BC.



$$\frac{Small\ Triangle}{Big\ Triangle} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

💡**TIP: ALWAYS DRAW A PICTURE FIRST IF THEY DON'T GIVE YOU ONE!**

PRACTICE PROBLEMS

Question 1:

Triangles ABC and DEF are similar, where $\overline{AB}$ corresponds to $\overline{DE}$. Triangle ABC has a perimeter of 90, and triangle DEF has a perimeter of 180. The length of AB is 4. What is the length of DE?

A) 1
B) 2
C) 8
D) 16

Question 2:

Triangles TUV and XYZ are similar, where T corresponds to X, and U corresponds to Y. The measure of angle T is $32°$, and $TU = 6$. Which of the following statements must be true?

I. The measure of angle X is $32°$.
II. $XY = 6a$, where a is a whole number.

A) I only
B) II only
C) I and II
D) Neither I nor II

Question 3:

Quadrilateral $W'X'Y'Z'$ is similar to quadrilateral $WXYZ$, where W, X, Y, Z correspond to W', X', Y', Z', respectively. The measure of angle W is $50°$, the measure of angle X is $80°$, and the measure of angle Y is $100°$. The length of each side of $W'X'Y'Z'$ is 4 times the length of each corresponding side of $WXYZ$. What is the measure of angle W'?

A) $50°$
B) $80°$
C) $130°$
D) $200°$

Question 4:

For each of two rectangles, the ratio of the rectangle's length to its width is $12:6$. The width of rectangle K is 18 times the width of rectangle L. How does the length of rectangle K compare to the length of rectangle L?

A) The length of rectangle K is 0.5 times the length of rectangle L.
B) The length of rectangle K is 2 times the length of rectangle L.
C) The length of rectangle K is 18 times the length of rectangle L.
D) The length of rectangle K is 36 times the length of rectangle L.

Question 5:

For a certain triangle, the ratio of its height to the length of its base is $16 : 12$. If the length of the base of the triangle increases by 9 units, how must the height change to maintain this ratio?

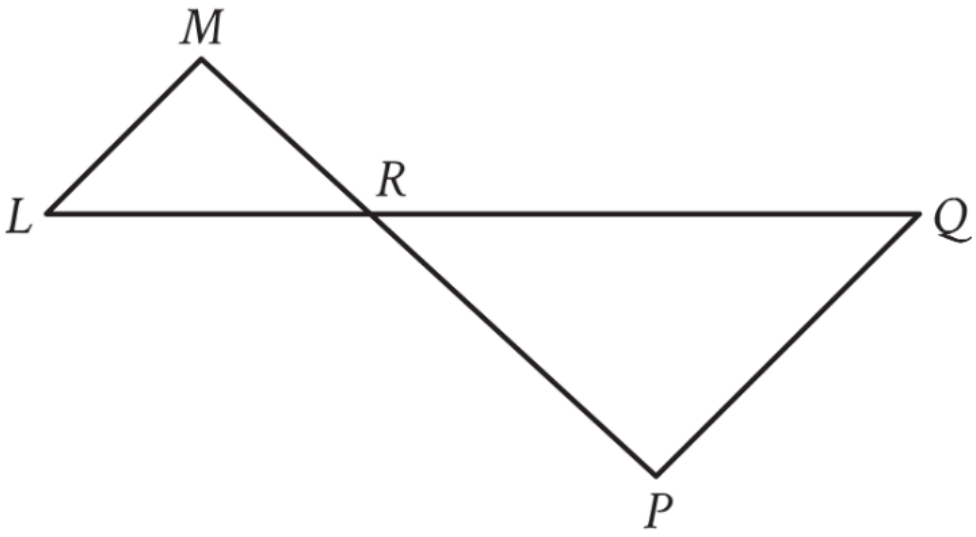
A) It must decrease by 9 units.

B) It must increase by 9 units.

C) It must increase by 12 units.

D) It must decrease by 12 units.

Question 6:



Note: Figure not drawn to scale.

In the figure, LQ intersects $M'P$ at point R, and LM is parallel to PQ. The lengths of MR, LR, and RP are 6, 8, and 14, respectively. What is the length of LQ?

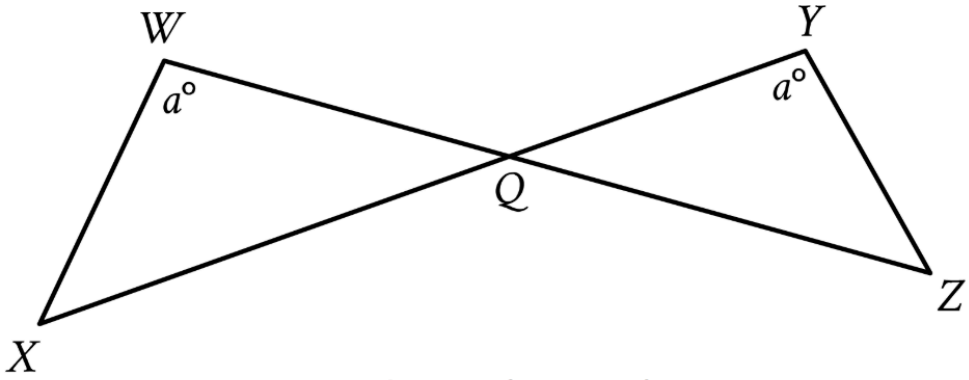
A) $\frac{24}{7}$

B) $\frac{80}{7}$

C) $\frac{56}{3}$

D) $\frac{80}{3}$

Question 7:

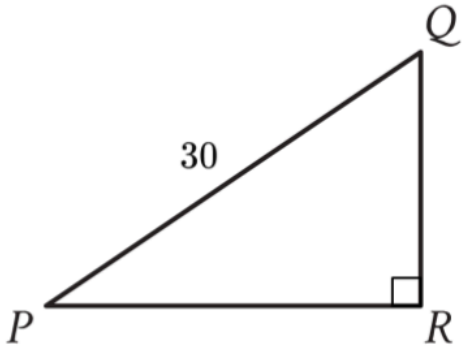


Note: Figure not drawn to scale.

In the figure shown, WZ and XY intersect at point Q. The segments have the following lengths: $YQ = 20$, $WQ = 50$, $WX = 60$, and $XQ = 90$. What is the length of YZ?

A) 24
B) 30
C) 45
D) 60

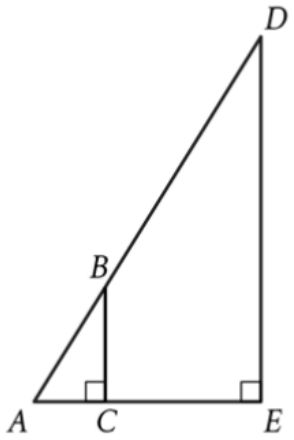
Question 8:



Note: Figure not drawn to scale.

In $\triangle PQR$, the length of $\overline{PR}$ is 10 less than the length of $\overline{PQ}$. Point S (not shown) lies on $\overline{PQ}$ such that RS is perpendicular to $\overline{PQ}$. What is the value of $\frac{RQ}{RS}$?

Question 9:



Note: Figure not drawn to scale.

In the figure shown, $AB = \sqrt{185}$ units, $AC = 4$ units, and $CE = 10$ units. What is the area, in square units, of triangle ADE?

Question 10:

In isosceles triangle LMN, sides $\overline{LM}$ and $\overline{LN}$ are congruent. Point P divides side $\overline{MN}$ such that the length of MP is $\frac{4}{9}$ of the length of $\overline{MN}$. Point Q lies on $\overline{LM}$ and point R lies on $\overline{LN}$ such that when segments $\overline{PQ}$ and $\overline{PR}$ are drawn, $\angle MQP$ is congruent to $\angle NRP$. If the length of MQ is 60, what is the length of NR?

A) 48
B) 75
C) 108
D) 135

Question 11:

Triangles **PQR** and **LMN** are graphed in the *xy*-plane. Triangle **PQR** has vertices **P**, **Q**, and **R** at $(4, 5)$, $(4, 7)$, and $(6, 5)$, respectively. Triangle **LMN** has vertices **L**, **M**, and **N** at $(4, 5)$, $(4, 7 + k)$, and $(6 + k, 5)$, respectively, where k is a positive constant. If the measure of $\angle Q$ is $t°$, what is the measure of $\angle N$?

A) $(90 - (t - k))°$
B) $(90 - (t + k))°$
C) $(90 - t)°$
D) $(90 + k)°$

GEOMETRY & TRIGONOMETRY CONCEPT #3: SIMILAR FIGURES ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. C**
- ☐ **2. A** (the scale factor doesn't have to be a whole number!)
- ☐ **3. A**
- ☐ **4. C**
- ☐ **5. C**
- ☐ **6. C**
- ☐ **7. A**
- ☐ **8. 1.5**
- ☐ **9. 637/2 or 318.5**
- ☐ **10. B**
- ☐ **11. C**

Concept #4: Triangle Proofs

WHAT YOU NEED TO KNOW

Triangle Inequality Theorem

The sum of **any two** sides of a triangle must be **greater than** the length of the third side.

Ways to Prove Congruency:

1._____

2. _____

3. _____

4. _____

5. _____

Ways to Prove Similarity:

1.

or

2. Any of the above options for congruency, provided you know that the sides are *in proportion* to each other.

PRACTICE PROBLEMS

Question 1:

The triangle inequality theorem states that the sum of any two sides of a triangle must be greater than the length of the third side. If a triangle has side lengths of 9 and 15, which inequality represents the possible lengths, x, of the third side of the triangle?

A) $6 \leq x \leq 24$
B) $6 < x < 24$
C) $x < 6$ or $x > 24$
D) $x \leq 6$ or $x \geq 24$

Question 2:

In triangle ABC, the measure of angle A is $35°$ and the measure of angle B is $75°$. In triangle XYZ, the measure of angle X is $35°$ and the measure of angle Y is $75°$. Which of the following additional pieces of information is needed to determine whether triangle ABC is similar to triangle XYZ?

A) The measure of angle C
B) The measure of angle Z
C) The measure of angle C and the measure of angle Z
D) No additional information is needed

Question 3:

In triangle FGH and triangle JKL, FG and JK are each equal to 9 yards, and angles F and J each have measure $42°$. Which additional piece of information is sufficient to prove that triangle FGH is congruent to triangle JKL?

A) The measures of angles G and H are equal.
B) The lengths of sides GH and KL are equal.
C) The lengths of sides FH and JL are equal.
D) No additional information is necessary to prove that the two triangles are congruent.

Question 4:

In triangle ABC, point D lies on BC such that $AD \perp BC$. Which additional piece of information is NOT sufficient to prove that triangle ABD is congruent to triangle ACD?

A) ABC is an isosceles triangle, with base BC
B) ABC is a right triangle, with hypotenuse BC
C) AD bisects angle A
D) AD bisects side BC

Question 5:

Two side lengths of a triangle are $n+2$ and n^2, where n is a positive integer. Which of the following could be the length of the third side?

A) 1
B) $n-n^2$
C) n^2+n+2
D) n^2+n+3

Question 6:

In triangle QRS and triangle TUV, the measures of angles R and U are each $45°$. The lengths of RS and UV are each 12 meters, and

$$\frac{RS}{RQ} = \frac{UV}{TU}.$$

Which additional piece of information would be necessary to prove that triangle QRS is congruent to triangle TUV?

A) The lengths of QR and TV are each 9 meters.
B) The measures of angles S and V are each $60°$.
C) The measures of angles Q and T are each $90°$.
D) No additional information is necessary.

Question 7:

In trapezoid $ABCD$, $AB \parallel CD$. Segments AC and BD are drawn, intersecting at point E. Which additional piece of information would be necessary to prove that $\triangle ABE$ is similar to $\triangle CDE$?

A) $ABCD$ is an isosceles trapezoid ($AD = BC$).
B) $ABCD$ is a right trapezoid (A and D are right angles).
C) AC is perpendicular to BD.
D) No additional information is necessary.

Question 8:

For triangle GHJ and triangle KLM, angles G and K each measure $40°$, $HJ = 28$, and $LM = 8$. Which additional piece(s) of information is(are) sufficient to prove that triangle GHJ is similar to triangle KLM?

I. The length of GH is $\frac{7}{2}$ the length of KL.
II. The length of GJ is $\frac{7}{2}$ the length of KM.

A) I is sufficient, but II is not.
B) II is sufficient, but I is not.
C) I is sufficient, and II is sufficient.
D) Neither I nor II is sufficient.

Question 9:

In triangle BCD and triangle FGH, sides BC and FG each have a side length of 7 centimeters, and angles B and F each have an angle measure of $130°$. Which of the following additional pieces of information is(are) sufficient to prove that triangle BCD is congruent to triangle FGH?

I. The measures of angles C and D are equal.

II. The lengths of sides BD and FH are equal.

III. The lengths of sides CD and GH are equal.

A) I is sufficient and III is sufficient, but II is not.

B) II is sufficient and III is sufficient, but I is not.

C) I is sufficient, but II and III are not.

D) II is sufficient, but I and III are not.

GEOMETRY & TRIGONOMETRY CONCEPT #4: TRIANGLE PROOFS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. B**
- ☐ **2. D**
- ☐ **3. C**
- ☐ **4. B** (A works because △ABC's base angles are congruent)
- ☐ **5. A** (specifically, only when n=2)
- ☐ **6. D** (given that 12/RQ = 12/TU, cross-multiply and simplify to get RQ = TU, which is enough for SAS)
- ☐ **7. D** (AB || CD is all you need – think transversals)
- ☐ **8. D** (you can use any congruence method for similarity, but both I and II run into an invalid SSA setup)
- ☐ **9. D** (statement III has that SSA issue again!)

Concept #5: Trigonometry

WHAT YOU NEED TO KNOW

SOH, CAH, TOA

Sin = —

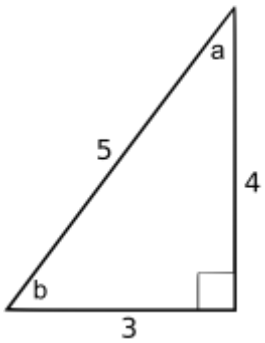
Cos = —

Tan = —

Complimentary Angles Rule

If $a + b = 90°$, then _____

Let's illustrate with an example:



sin a = _____

cos b = _____

If sin K = 0.6, what does that mean?

Write 0.6 as a fraction or ratio! _____

Opposite side = _____

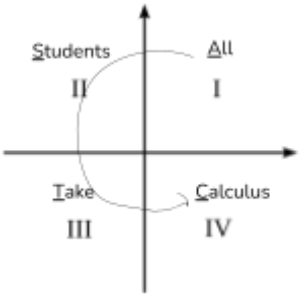
Hypotenuse = _____

Pythagorean Theorem _____
(to find the missing side of a right triangle)

a - leg
b - leg
c - hypotenuse

Recognizing a 3,4,5 right triangle:

When are Sin, Cosine and Tangent Positive?

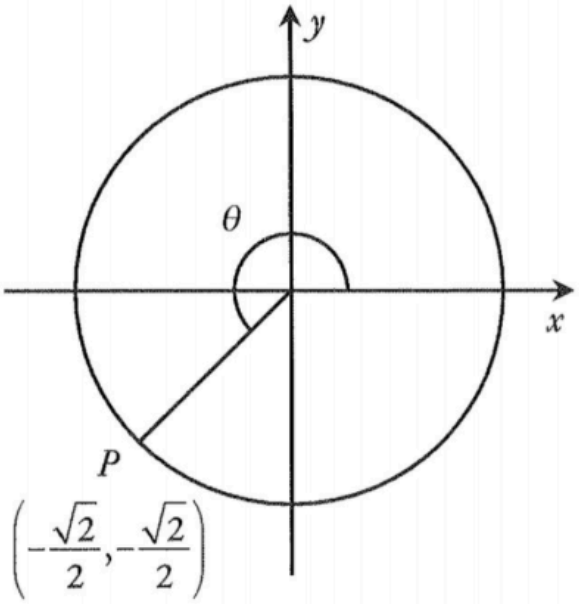


Special Right Triangles (Available on Your Cheat Sheet) - you can use these instead of memorizing the unit circle!

1. Draw a picture of a 30, 60, 90

2. Draw a picture of a 45, 45, 90

Example:



The angle θ on the unit circle (shown above), formed when the segment that connects the origin to point $P\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$ meets the positive x-axis, can be written as $\frac{10\pi}{a}$ radians. What is the value of a?

PRACTICE PROBLEMS

Question 1:

In right triangle DEF, the sum of the measures of angle D and angle E is 90 degrees. The value of $\sin(D)$ is $\frac{15\sqrt{133}}{11}$. What is the value of $\cos(E)$?

A) $\frac{20\sqrt{133}}{15}$

B) $\frac{15\sqrt{133}}{11}$

C) $\frac{11\sqrt{133}}{15}$

D) $\frac{15\sqrt{133}}{20}$

Question 2:

In isosceles right triangle RST, $RS = 23$ and $ST = 23$. What is the length of RT?

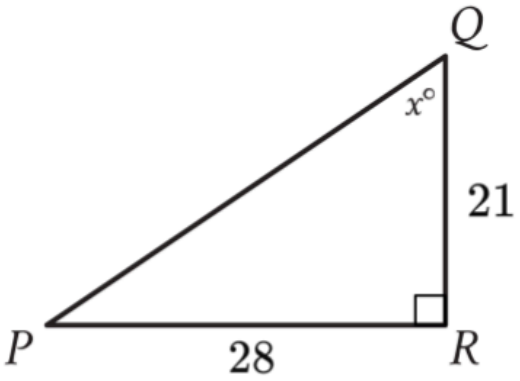
A) $\sqrt{23}$

B) $\sqrt{46}$

C) $23\sqrt{2}$

D) $46\sqrt{2}$

Question 3:

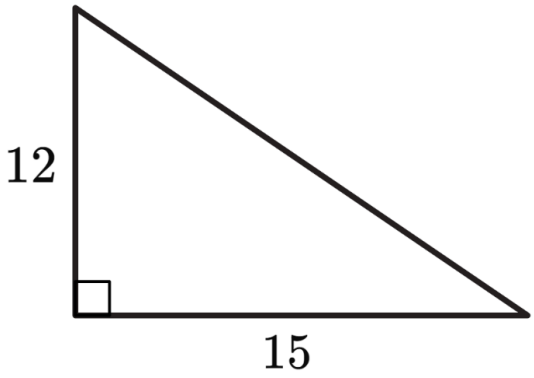


Note: Figure not drawn to scale.

In the right triangle shown, the value of $\tan x = \frac{d}{21}$. What is the value of d?

A) 21
B) 28
C) 35
D) 42

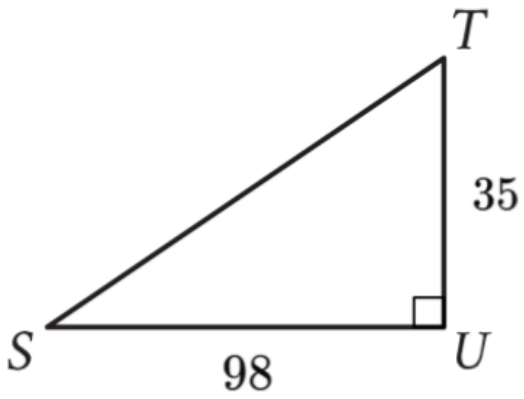
Question 4:



Note: Figure not drawn to scale.

What is the length of the triangle's hypotenuse?

Question 5:

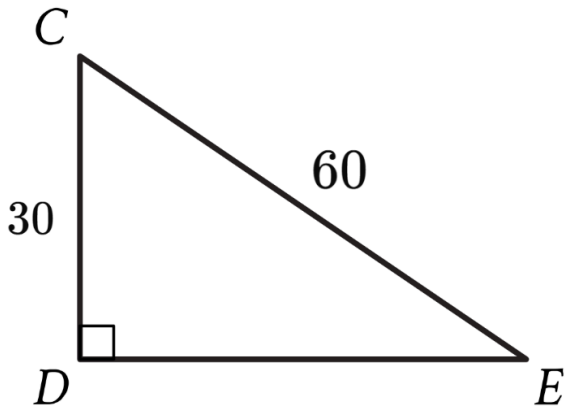


Note: Figure not drawn to scale.

Triangle STU shown is a right triangle. Which of the following has the same value as $\sin S$?

A) $\cos S$
B) $\cos T$
C) $\tan S$
D) $\tan T$

Question 6:



Note: Figure not drawn to scale.

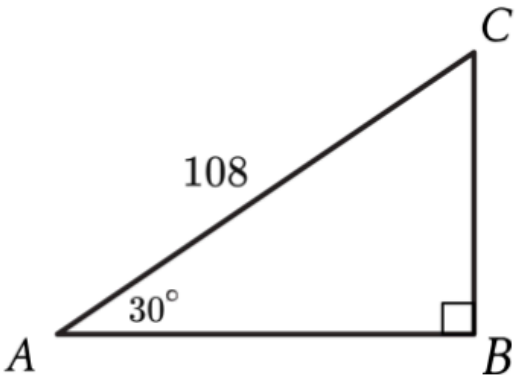
In $\triangle CDE$ as shown, what is the value of $\cos E$?

Question 7:

Isosceles right triangle LMN has a perimeter of $62 + 62\sqrt{2}$ inches. What is the length, in inches, of one of the legs of this triangle?

A) 31
B) $31\sqrt{2}$
C) 62
D) $62\sqrt{2}$

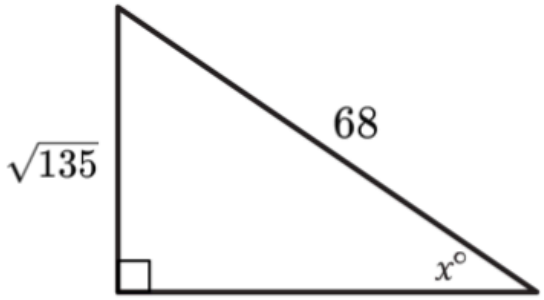
Question 8:



Note: Figure not drawn to scale.

In $\triangle ABC$ as shown, what is the value of $\tan C$?

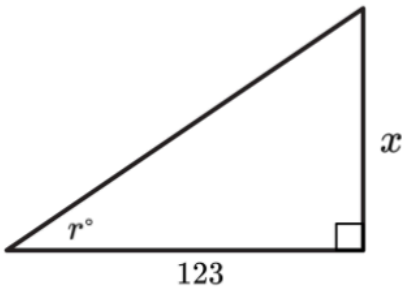
Question 9:



Note: Figure not drawn to scale.

What is the value of $\cos x$?

Question 10:

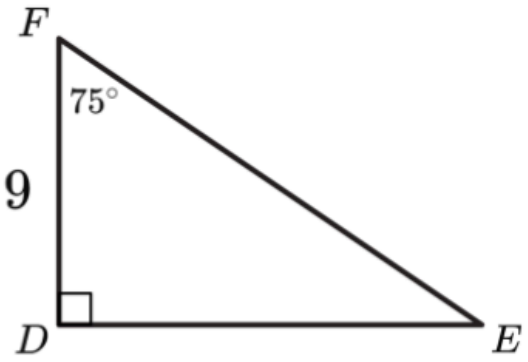


Note: Figure not drawn to scale.

In the triangle shown, $r = 30$. What is the value of x?

A) 41
B) $41\sqrt{3}$
C) $123\sqrt{3}$
D) 246

Question 11:



Note: Figure not drawn to scale.

Which expression represents the length of line segment EF?

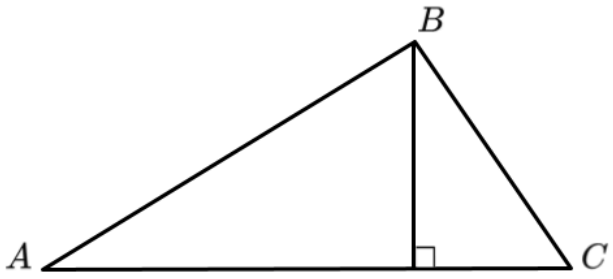
A) $9 \cos 75°$

B) $\frac{9}{\cos 75°}$

C) $9 \sin 75°$

D) $\frac{9}{\sin 75°}$

Question 12:



Note: Figure not drawn to scale.

In the figure shown, the measure of $\angle A$ is $29°$. The length of $\overline{AB}$ is 19 units and the length of $\overline{AC}$ is 28 units. What is the area, in square units, of $\triangle ABC$?

A) $266 \sin 29°$

B) $532 \sin 29°$

C) 266

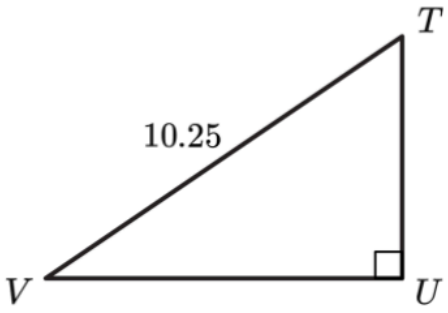
D) 532

Question 13:

In right triangle XYZ, angles X and Y are acute, side XZ has a length of 25.5, and $\tan(Y) = 5$. What is the length of side ZY, rounded to the nearest tenth?

A) 5.1
B) 26.0
C) 127.5
D) 130.0

Question 14:



Note: Figure not drawn to scale.

In right triangle TUV, the tangent of $\angle T$ is 0.225. What is the length of TU?

A) 2.25
B) 4.5
C) 10
D) 20

Question 15:

In $\triangle JKL$, $\angle K$ is a right angle and the length of KL is 28 feet. If $\cos J = 0.96$, what is the length, in feet, of JK?

A) 24
B) 25
C) 96
D) 100

Question 16:

$$\frac{1}{9}\sin(90 - x + p) \cdot \cos(q) + \cos(x - p) \cdot \sin(90 - q)$$

In the given expression, p and q are constants such that $\sin(p) = 0.84$, $\cos(q) = 0.75$, $0 < p < 90$, and $0 < q < 90$. What is the value of the expression when $x = 90$?

Question 17:

In triangle KLM, angle L is a right angle, the measure of angle K is $33°$, and the length of LM is 19 centimeters. If the area, in square centimeters, of triangle KLM can be represented by the expression $n \cdot \tan(57°)$, where n is a constant, what is the value of n?

GEOMETRY & TRIGONOMETRY CONCEPT #5: TRIGONOMETRY ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. B**
- ☐ **2. C**
- ☐ **3. B**
- ☐ **4. 19.21 (or 19.20)**
- ☐ **5. B**
- ☐ **6. 0.866**
- ☐ **7. B**
- ☐ **8. 1.732**
- ☐ **9. 0.985, .9853, or 67/68**
- ☐ **10. B**
- ☐ **11. B**
- ☐ **12. A**
- ☐ **13. A**
- ☐ **14. C**
- ☐ **15. C**
- ☐ **16. 0.7**
- ☐ **17. 180.5**

Concept #6: Circles

WHAT YOU NEED TO KNOW

1) **Circumference Formula:** _____

2) **Area Formula:** _____

Note: Both 1 and 2 are on your math reference table if you forget them.

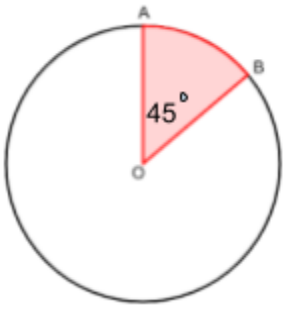
3) a. **Convert radians to degrees :**

b. **Convert degrees to radians:**

4) # of degrees in a circle: _____ and # of radians in a circle: _____

5) Finding the fraction of a segment:

Take the central angle and divide it by total degrees in a circle.



$\frac{45}{\quad} = \_\_$

Thus, everything about this segment is _____ of the whole.

6) Completing the Square

Yes, you can use Desmos to find the center and the radius of a circle that is not in standard form.. unless you can't (see problem #11). It is still important to know the process!

$$x^2 + y^2 - 4x + 6y = 12$$

Standard Form: $(x \quad - \quad)^2 + (y \quad + \quad)^2 =$

Center: (,)

Radius:

PRACTICE PROBLEMS

Question 1:

A circle has a circumference of 10π centimeters. What is the diameter, in centimeters, of the circle?

Question 2:

A circular park has a radius of 8 miles. What is the area, in square miles, of the park?

A) 4π

B) 8π

C) 16π

D) 64π

Question 3:

Circle R has a radius of 2 units. Circle S has an area of 9π square units. What is the total area, in square units, of circles R and S?

A) 11π

B) 13π

C) 83π

D) 85π

Question 4:

A square is inscribed in a circle such that the circumference of the circle touches each corner of the square. The length of one side of the square is 66 units. What is the area, in square units, of the circle?

A) 2178π

B) 4356π

C) 6534π

D) 8712π

Question 5:

$$p(x) = \frac{3}{10}(2\pi x)$$

The function p gives the partial length, in inches, of the circumference of a circle with a radius of x inches. According to this function, for each increase of 14 inches in the length of the radius, this partial length of the circumference increases by $n\pi$ inches. What is the value of n?

Question 6:

Points G and H lie on a circle with center C. The measure of arc GH is $18°$, and the length of this arc is 40 millimeters. What is the circumference, in millimeters, of the circle?

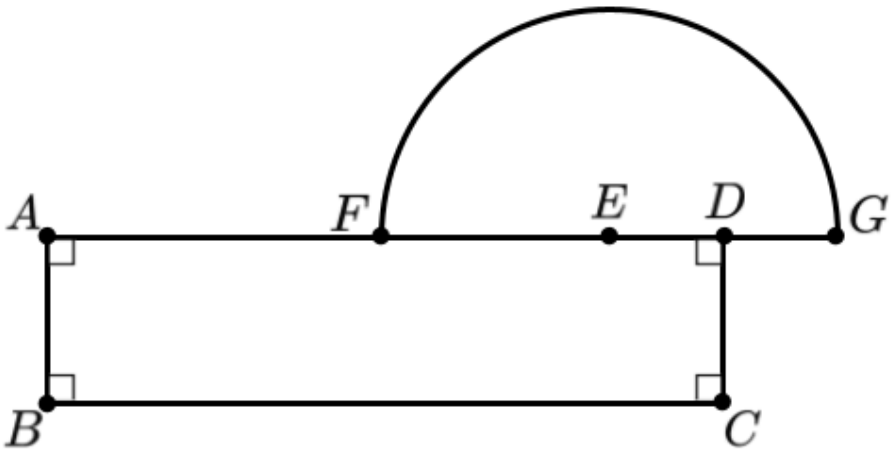
A) 200

B) 400

C) 800

D) 1600

Question 7:



Note: Figure not drawn to scale.

In the figure shown, $\overline{FG}$ is the diameter and E is the center of the semicircle. Point D is the midpoint of $\overline{EG}$, and point F is the midpoint of $\overline{AD}$. If $CD = 4$ feet and $ED = 3$ feet, which expression represents the total area of this figure, in square feet?

A) $48 + 18\pi$
B) $48 + 36\pi$
C) $72 + 18\pi$
D) $72 + 36\pi$

Question 8:

Point A lies on a unit circle in the xy-plane and has coordinates $(0, 1)$. Point O is the center of the circle and has coordinates $(0, 0)$. Point B also lies on the circle and has coordinates $(-c, c)$, where c is a positive constant. Which of the following could be the positive measure of angle AOB, in radians?

A) $\frac{31\pi}{2}$
B) $\frac{33\pi}{2}$
C) $\frac{31\pi}{4}$
D) $\frac{33\pi}{4}$

Question 9:

A circle has diameters WY and XZ. The circumference of the circle is 36π, and the length of arc ZW is 3 times the length of arc WX. What is the length of arc XY?

A) $\frac{9\pi}{2}$

B) 9π

C) $\frac{27\pi}{2}$

D) 27π

Question 10:

Point P lies on a unit circle in the xy-plane and has coordinates $(1, 0)$. Point O is the center of the circle and has coordinates $(0, 0)$. Point Q also lies on the unit circle, such that the measure of angle POQ is $\frac{155\pi}{6}$ radians. If the coordinates of point Q are (m, n), where m and n are constants, what is the value of m?

A) $\frac{1}{2}$

B) $\frac{\sqrt{3}}{2}$

C) $-\frac{1}{2}$

D) $-\frac{\sqrt{3}}{2}$

Question 11:

$$x^2 + y^2 - 8x + 2y - 16n = 0$$

This is the equation of Circle A.

Circle B has the same center as Circle A but has **twice the diameter** of Circle A and passes through the point $(14, 7)$.

What is the value of n?

Question 12:

$$x^2 + y^2 - 16x - 4y - 14n = 0$$

This is the equation of circle A.
Circle B has the same center, but twice the diameter of circle A and passes through the point $(10, 12)$. What is the value of n?

GEOMETRY & TRIGONOMETRY CONCEPT #6: CIRCLES ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

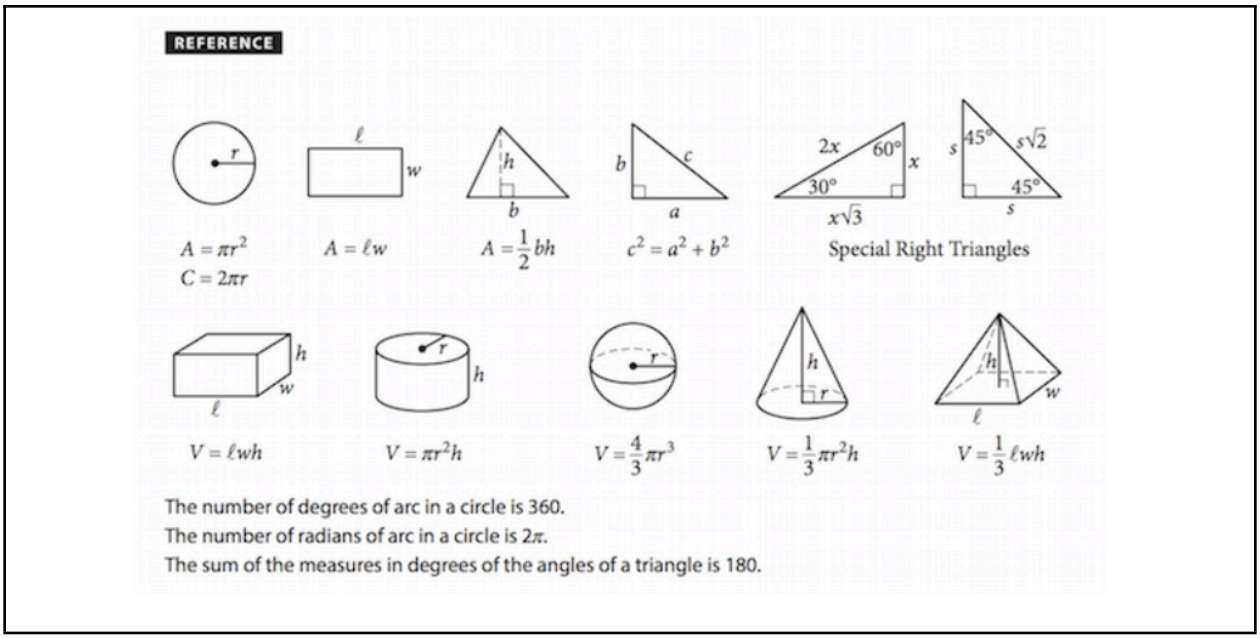
Date completed: _____

- ☐ **1. 10**
- ☐ **2. D**
- ☐ **3. B**
- ☐ **4. A**
- ☐ **5. 8.4 or 42/5**
- ☐ **6. C**
- ☐ **7. C**
- ☐ **8. D**
- ☐ **9. C**
- ☐ **10. D**
- ☐ **11. 1.5**
- ☐ **12. -3**

Concept #7: Volume/Surface Area

WHAT YOU NEED TO KNOW

Use your math reference table in BlueBook for formulas.



How to Get Volume of a Triangular Prism (not on reference table):

_____

3. Sometimes you need to make up your own numbers to determine a relationship to get what they want you to get (i.e. if they ask you to cut the radius in half and double the height, but they don't give you the original dimensions).

4. They may ask about surface area instead.

Surface area of a rectangular prism formula = _____

PRACTICE PROBLEMS

Question 1:

A sphere has a radius of 0.5 meters. What is the volume, in cubic meters, of the sphere?

A) $\frac{\pi}{6}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{3}$

D) $\frac{2\pi}{3}$

Question 2:

A right circular cone has a height of 9 mm and a base with a radius of 5 mm. What is the volume, in mm^3, of this cone?

A) 25π

B) 45π

C) 75π

D) 135π

Question 3:

A cylinder has a diameter of 12 units and a height of 5 units. What is the volume, in cubic units, of the cylinder?

A) 36π
B) 60π
C) 180π
D) 720π

Question 4:

The length of each edge of a box is 2.5 feet. Each side of the box is in the shape of a square. The box does not have a lid. What is the exterior surface area, in square **inches**, of this box without a lid?
$(1 \text{ foot} = 12 \text{ inches})$

Question 5:

A hemisphere is half of a sphere. If a hemisphere has a radius of 12 centimeters, which of the following is closest to the volume, in cubic centimeters, of this hemisphere?

A) 200
B) 800
C) 2700
D) 3600

Question 6:

A cube has a volume of $238,328$ cubic units. What is the surface area, in square units, of the cube?

Question 7:

The circumference of the base of a right circular cylinder is 8π meters, and the height of the cylinder is 10 meters. What is the volume, in cubic meters, of the cylinder?

A) 40π
B) 80π
C) 160π
D) 640π

Question 8:

A certain alloy has a density of 44.8 grams per cubic centimeter. A scientist receives a sample of this alloy in the shape of a cube. The length of one edge of this cube is 1.75 centimeters. What is the mass of the cube, in grams?

Question 9:

A right rectangular prism has a base area of $45r$ ft². The length of the base of the rectangular prism is 9 ft, and the height of the rectangular prism is 6 ft. Which expression represents the surface area, in ft², of the right rectangular prism?

A) $108 + 90r$
B) $150 + 108r$
C) $60r + 108$
D) $150r + 108$

Question 10:

The height of a right circular cylinder is 7 inches longer than its radius, and the surface area of the cylinder is 294π square inches. What is the cylinder's height, in inches?

A) 7
B) 14
C) 21
D) 49

Question 11:

The volume of a right rectangular prism with a square base is 3.757 cubic centimeters. If the area of each of the four lateral faces is 221 square millimeters, what is the height, in millimeters, of the prism?

A) 13
B) 17
C) 221
D) 289

Question 12:

A right circular cone has a volume of 1024π cubic inches and the area of its base is 256π square inches. What is the slant height, in inches, of this cone?

A) 4
B) 12
C) 20
D) 28

Question 13:

A cube has an edge length of 60 meters. A solid sphere with a radius of 30 meters is inside the cube, such that the sphere touches the center of each face of the cube. The volume of space in the cube not taken up by the sphere can be written as $1000\,k$ cubic meters, where k is a constant. To the nearest whole number, what is the value of k?

A) 90
B) 103
C) 108
D) 113

GEOMETRY & TRIGONOMETRY CONCEPT #7: VOLUME/SURFACE AREA ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. A**
- ☐ **2. C**
- ☐ **3. C**
- ☐ **4. 4500**
- ☐ **5. D**
- ☐ **6. 23064**
- ☐ **7. C**
- ☐ **8. 240.1**
- ☐ **9. D**
- ☐ **10. B**
- ☐ **11. A**
- ☐ **12. C**
- ☐ **13. B**

Concept #8: Scalar Multiples in Different Dimensions

It is important to recognize what dimension you are working with when given a question about similar figures and then adjust the scalar multiple to work in other dimensions.

If a length of the sides of one similar shape is 4 times bigger, then

- Area = (___ ^{2}) = _____ times bigger
- Volume = (___ ^{3}) = _____ times bigger

EXAMPLE

Question:

Two rectangular prisms are similar in shape.

- Prism 1 has a surface area of 64 square units and a volume of 25.4 cubic units.
- Prism 2 has a surface area of 1600 square units.

What is the volume of Prism 2?

PRACTICE PROBLEMS

Question 1:

The length of one side of square Q is 5 times the length of one side of square R. The area of square R is 16 square centimeters. What is the area, in square centimeters, of square Q?

A) 25
B) 80
C) 100
D) 400

Question 2:

Circle J has a radius of $6z$, and Circle K has a radius of $156z$, where z is a positive constant. The area of Circle K is how many times the area of Circle J?

Question 3:

Rectangles $QRST$ and $WXYZ$ are similar. The length of each side of $WXYZ$ is 4 times the length of the corresponding side of $QRST$. The area of $WXYZ$ is 80 square units. What is the area, in square units, of $QRST$?

A) 5
B) 20
C) 320
D) 1280

Question 4:

Triangles LMN and PQR are similar. Each side length of LMN is 3 times the corresponding side length of PQR. The area of LMN is 405 square units. What is the area, in square units, of PQR?

A) 45
B) 135
C) 1215
D) 3645

Question 5:

Square A has a side length of y meters. Square B has a perimeter that is 28 meters greater than the perimeter of square A. The function g gives the area of square B, in square meters. Which of the following defines g?

A) $g(y) = (y + 7)^2$
B) $g(y) = (y + 28)^2$
C) $g(y) = (y + 14)^2$
D) $g(y) = (4y + 28)^2$

Question 6:

A rectangular poster has an area of 1600 square centimeters. A copy of the poster is made in which the length and width of the original poster are each increased by 25%. What is the area of the copy, in square centimeters?

A) 100
B) 2000
C) 2400
D) 2500

Question 7:

Rectangle $ABCD$ is similar to rectangle $WXYZ$. The area of rectangle $ABCD$ is 486 square units, and the area of rectangle $WXYZ$ is 54 square units. The length of the longest side of rectangle $ABCD$ is 27 units. What is the length, in units, of the longest side of rectangle $WXYZ$?

A) 6
B) 9
C) 12
D) 18

Question 8:

A right circular cone has a radius of r and a height of h. A second right circular cone has a volume that is 432 times the volume of the first cone. Which of the following could represent the radius R, in terms of r, and the height H, in terms of h, of the second cone?

A) $R = 12r$ and $H = 6h$
B) $R = 12r$ and $H = 8h$
C) $R = 6r$ and $H = 12h$
D) $R = 8r$ and $H = 12h$

Question 9:

Right circular cylinder C has a volume of 125π cubic units, and right circular cylinder D has a volume of 15625π cubic units. The two cylinders are similar, and the radius of cylinder C is 5 units. The surface area of cylinder C is $x\pi$ square units, and the surface area of cylinder D is $y\pi$ square units, where x and y are constants. What is the value of $y - x$?

(The surface area of a right circular cylinder with radius r and height h is $2\pi r^2 + 2\pi rh$.)

Question 10:

Two spheres have radii in the ratio $12 : 13$. The difference in their surface areas is 400π square inches. If the volume of the smaller sphere can be written as $v\pi$, what is the value of v? *(Surface area of a sphere: $A = 4\pi r^2$)*

GEOMETRY & TRIGONOMETRY CONCEPT #8: SCALAR MULTIPLES IN DIFFERENT DIMENSIONS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. D**
- ☐ **2. 676**
- ☐ **3. A**
- ☐ **4. A**
- ☐ **5. A**
- ☐ **6. D**
- ☐ **7. B**
- ☐ **8. C**
- ☐ **9. 2400**
- ☐ **10. 18432**

Concept #9: Circle Problems that Aren't Really Circle Problems

WHAT YOU NEED TO KNOW

Most **"circle"** problems are really triangles problems in disguise.

Tangent = forms a _____ angle

Isosceles = _____ equal sides (think: two radii of a circle)

A right triangle inside of a right triangle = _____ triangles!

PRACTICE PROBLEMS

Question 1:

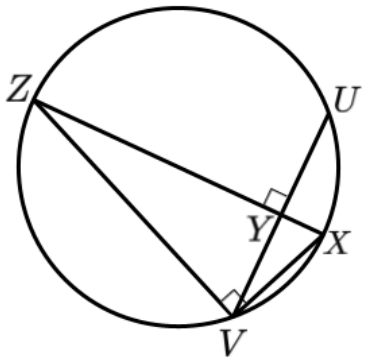
A circle has center G, and points M and N lie on the circle. Line segments MH and NH are tangent to the circle at points M and N, respectively. If the radius of the circle is 150 millimeters and the perimeter of quadrilateral GMHN is 3,500 millimeters, what is the distance, rounded to the nearest millimeter, between points G and H?

A. 150
B. 1,600
C. 1,607
D. 1,617

Question 2:

The perimeter of an equilateral triangle is 72 centimeters. The three vertices of the triangle lie on a circle. The radius of the circle is $w\sqrt{3}$ centimeters. What is the value of w?

Question 3:



Note: Figure not drawn to scale.

In the figure shown, points U, V, X, and Z lie on the circle, and $VX < VZ$. Segment XZ is the diameter of the circle and has length 150. Segment XZ is perpendicular to segment UV at point Y, and $VY = \sqrt{296}$. If $\frac{XY}{YZ} = k$, what is the value of k?

GEOMETRY & TRIGONOMETRY CONCEPT #9: CIRCLE PROBLEMS THAT AREN'T REALLY CIRCLE PROBLEMS ANSWER KEY

Directions: Check the box if you got the question correct. Any unchecked questions should be revisited at a later time.

Date completed: _____

- ☐ **1. C**
- ☐ **2. 8**
- ☐ **3. .0135 or 1/74**

Notes

Notes